Masses of Ground- & Excited-State Hadrons

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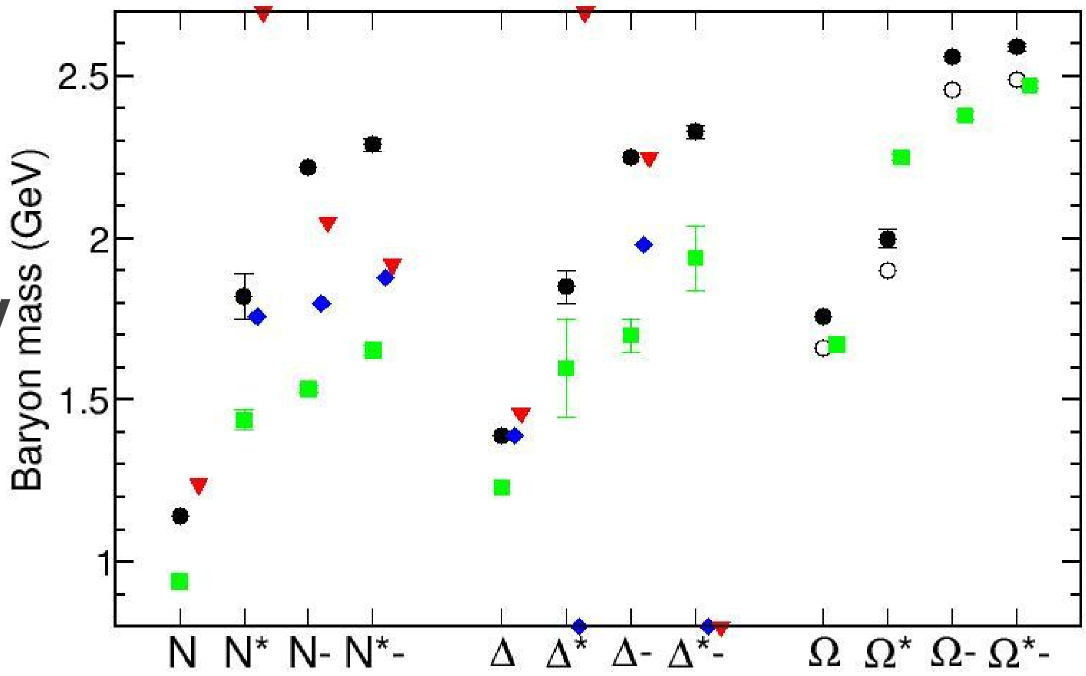
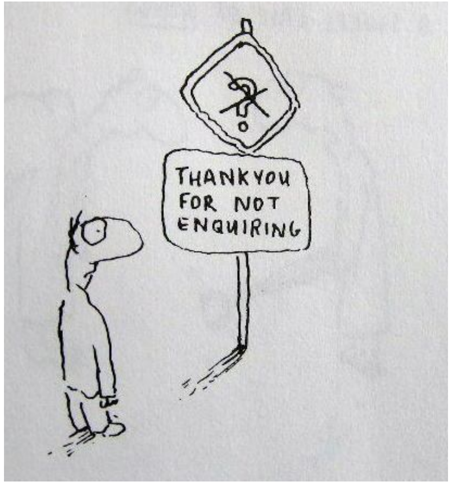


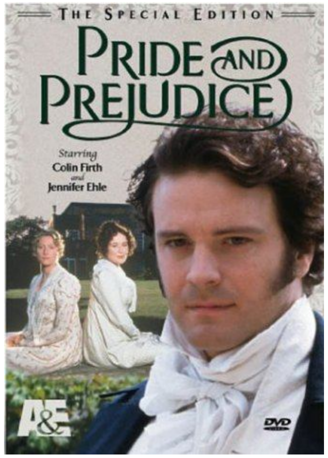
Masses of ground and excited-state hadrons
Hannes L.L. Roberts, Lei Chang, Ian C. Cloët and Craig D. Roberts
arXiv:1101.4244 [nucl-th] , to appear in *Few Body Systems*



QCD's Challenges

Understand emergent phenomena

- Quark and Gluon Confinement
  No matter how hard one strikes the proton,
  one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  Very unnatural pattern of bound state masses;
  e.g., Lagrangian (pQCD) quark mass is small but
  . . . no degeneracy between $J^P=+$ and $J^P=-$ *(parity partners)*
- Neither of these phenomena is apparent in QCD's Lagrangian
  Yet they are the dominant determining characteristics of real-world QCD.
- ***Both*** will be important herein
- QCD
  – Complex behaviour arises from apparently simple rules.



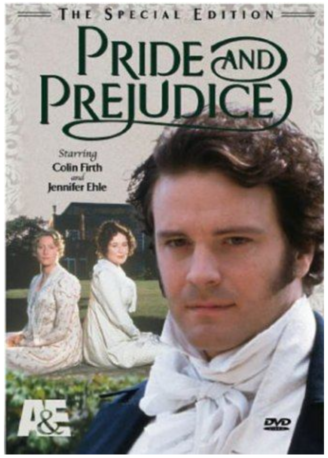
Universal Truths

- Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.

  Higgs mechanism is (*almost*) irrelevant to light-quarks.
- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach.

  Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.
- Confinement is expressed through a violent change of the propagators for coloured particles & can almost be read from a plot of a states' dressed-propagator.

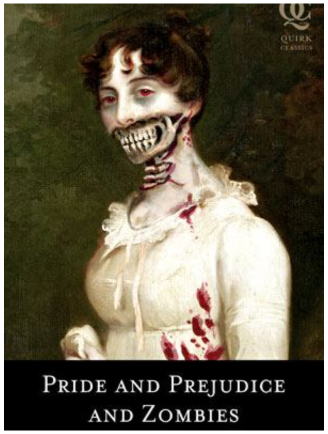
  It is intimately connected with DCSB.



Universal *Conventions*

- Wikipedia: (http://en.wikipedia.org/wiki/QCD_vacuum)

  "The QCD vacuum is the vacuum state of quantum chromodynamics (QCD). It is an example of a non-perturbative vacuum state, characterized by many non-vanishing condensates such as the gluon condensate or the quark condensate. These condensates characterize the normal phase or the confined phase of quark matter."



Universal Misapprehensions

- Since 1979, DCSB has commonly been associated *literally* with a spacetime-independent mass-dimension-three "vacuum condensate."
- Under this assumption, "condensates" couple directly to gravity in general relativity and make an enormous contribution to the *cosmological constant*

$$\Omega_{QCD-condensates} = 8\pi\, G_N \frac{\Lambda_{QCD}^4}{3H_0^2} \simeq 10^{46}$$

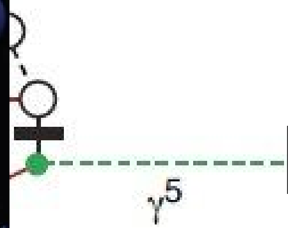
- Experimentally, the answer is

$$\Omega_{\text{cosm. const.}} = 0.76$$

- This mismatch is a bit of a problem.

Resolution?

- *Quantum Healing Central*:
  - "KSU physics professor [Peter Tandy] publishes groundbreaking research on inconsistency in Einstein theory."
- *Paranormal Psychic Forums:*
  - "Now Stanley Brodsky of the SLAC National Accelerator Laboratory in Menlo Park, California, and colleagues have found a way to get rid of the discrepancy. "People have just been taking it on faith that this quark condensate is present throughout the vacuum," says Brodsky.



Paradigm shift: In-Hadron Condensates

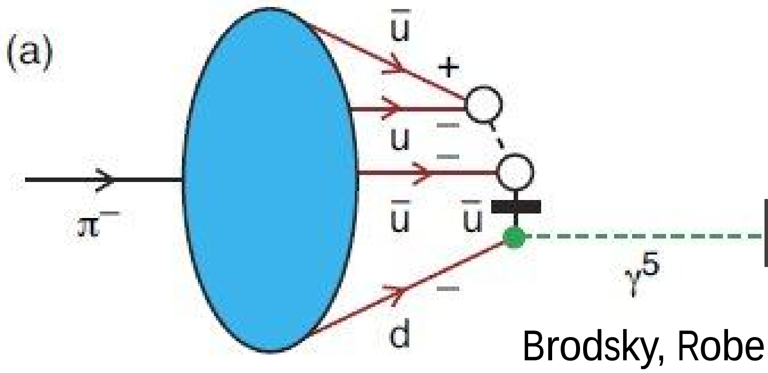
Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201
Brodsky and Shrock, arXiv:0905.1151 [hep-th], PNAS **108**, 45 (2011)

- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **GMOR**
    cf.

$$f_\pi^2 m_\pi^2 = -2\, m(\zeta) \langle \bar{q} q \rangle_0^\zeta$$

QCD

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$



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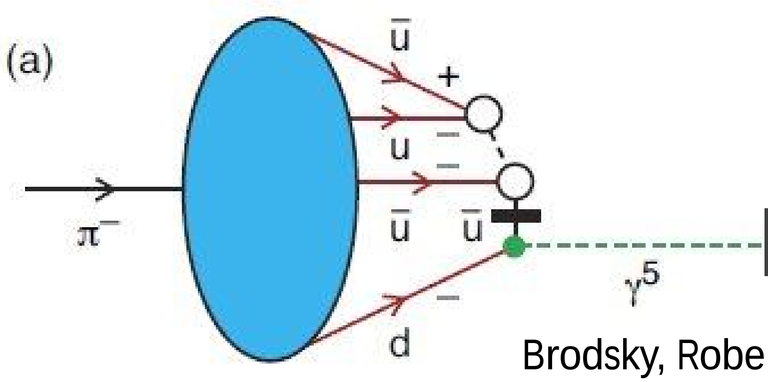
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **No qualitative difference between f_π and ρ_π**

$$
\begin{aligned}
i f_\pi P_\mu &= \langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle \\
&= Z_2(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4} i\gamma_5\gamma_\mu S(q_+)\Gamma_\pi(q;P)S(q_-),
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
i\rho_\pi &= -\langle 0|\bar{q}i\gamma_5 q|\pi\rangle \\
&= Z_4(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4}\gamma_5 S(q_+)\Gamma_\pi(q;P)S(q_-).
\end{aligned} \tag{6}
$$



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  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **No qualitative difference between f_π and ρ_π**
  - **And**

$$-\langle \bar{q} q \rangle_\zeta^\pi \equiv -f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m};\zeta).$$

Chiral limit

$$\kappa_\pi(0;\zeta) = -<\bar{q}q>_\zeta^0$$

Paradigm shift: In-Hadron Condensates

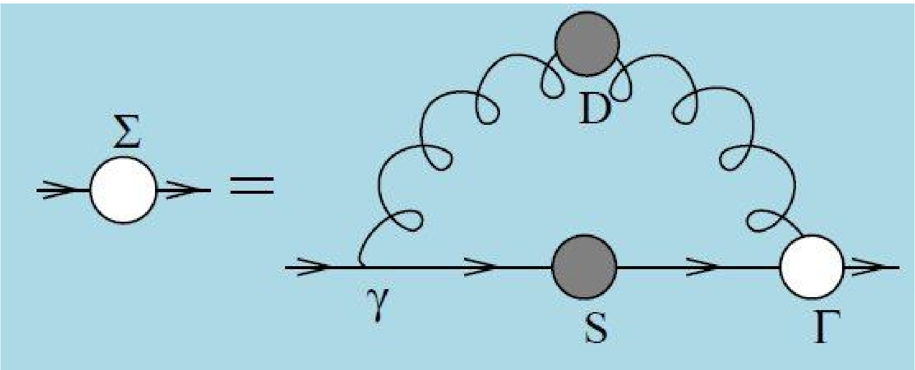
"Void that is truly empty
solves dark energy puzzle"
Rachel Courtland, New Scientist 4th Sept. 2010

"EMPTY space may really be empty. Though quantum theory suggests that a vacuum should be fizzing with particle activity, it turns out that this paradoxical picture of nothingness may not be needed. A calmer view of the vacuum would also help resolve a nagging inconsistency with dark energy, the elusive force thought to be speeding up the expansion of the universe."

$$\Omega_{QCD-condensates} = 8\pi G_N \frac{\Lambda_{QCD}^4}{3H_0^2} \simeq 10^{46}$$

Cosmological Constant:

- ✓ ***Putting QCD condensates back into hadrons reduces the mismatch between experiment and theory by a factor of 10^{46}***
- ✓ ***Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model***



QCD and Hadrons

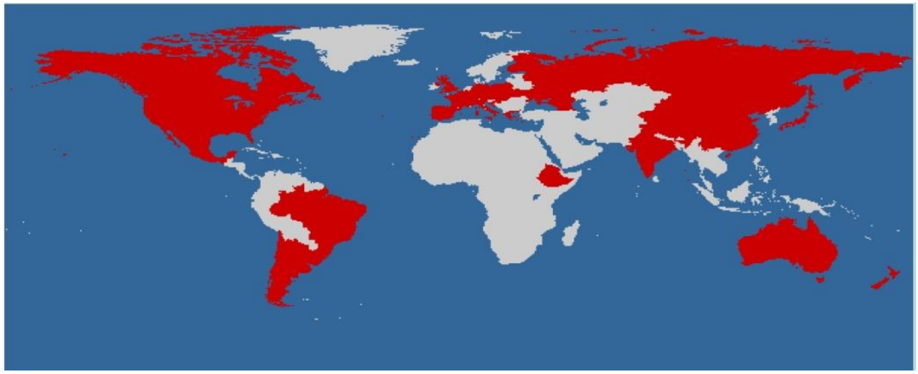
Gap equation

- Nonperturbative tools are needed
  - Quark models; Lattice-regularized QCD; Sum Rules; Generalised Parton Distributions

    "Theory Support for the Excited Baryon Program at the JLab 12- GeV Upgrade" – arXiv:0907.1901 [nucl-th]
- Dyson-Schwinger equations
  - Nonperturbative symmetry-preserving tool

    for the study of Continuum-QCD
- DSEs provide complete and compelling understanding of the pion as *both* a bound-state & *Nambu-Goldstone mode* in QCD

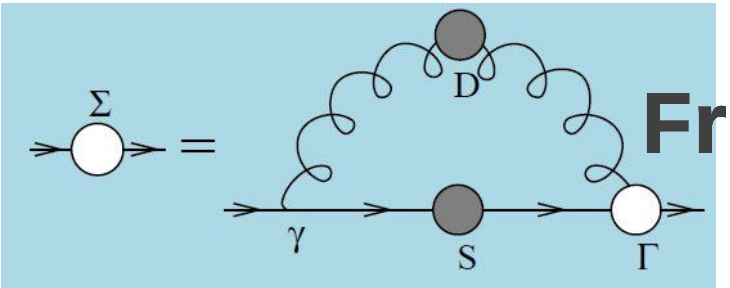
Pion mass and decay constant, P. Maris, C.D. Roberts and P.C. Tandy nucl-th/9707003, Phys. Lett. B**20** (1998) 267-273



Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory . . . Materially Reduces Model-Dependence … Statement about long-range behaviour of quark-quark interaction
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    - Coloured objects not detected, *Not detectable?*

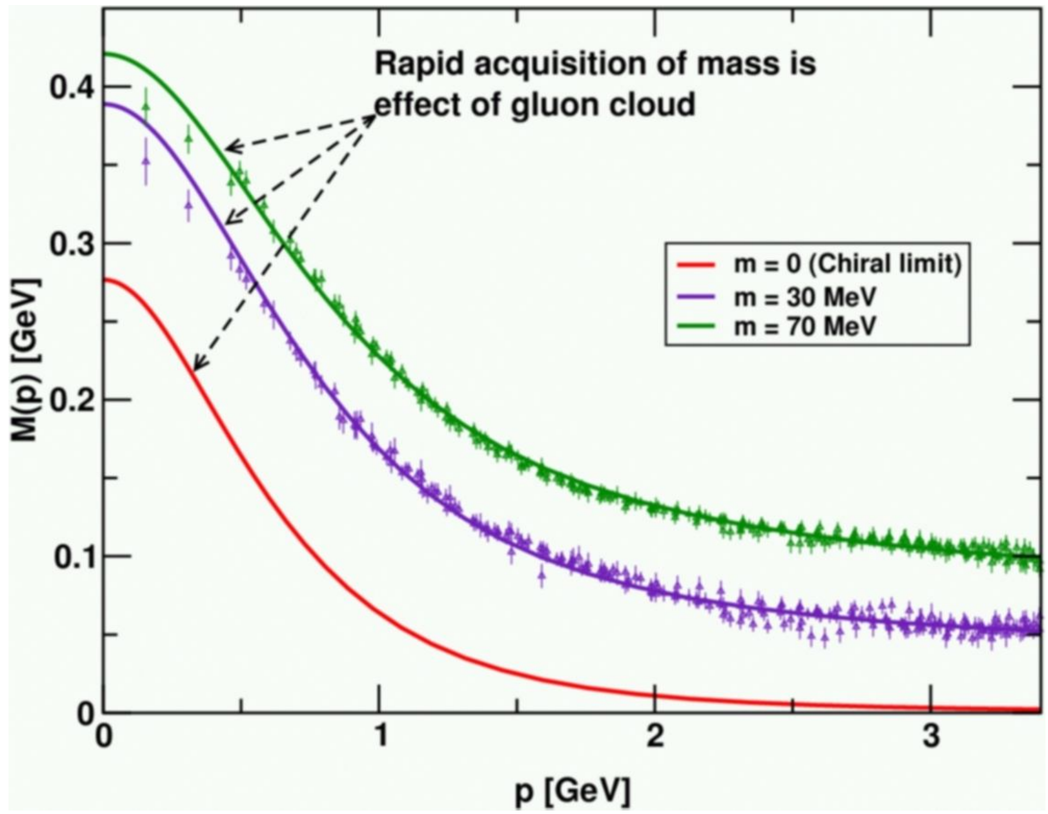
- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling
- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling

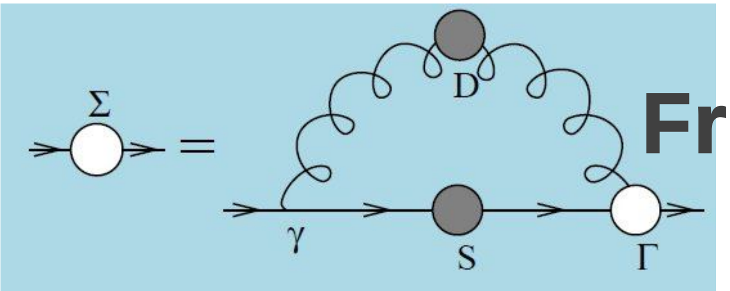


Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.





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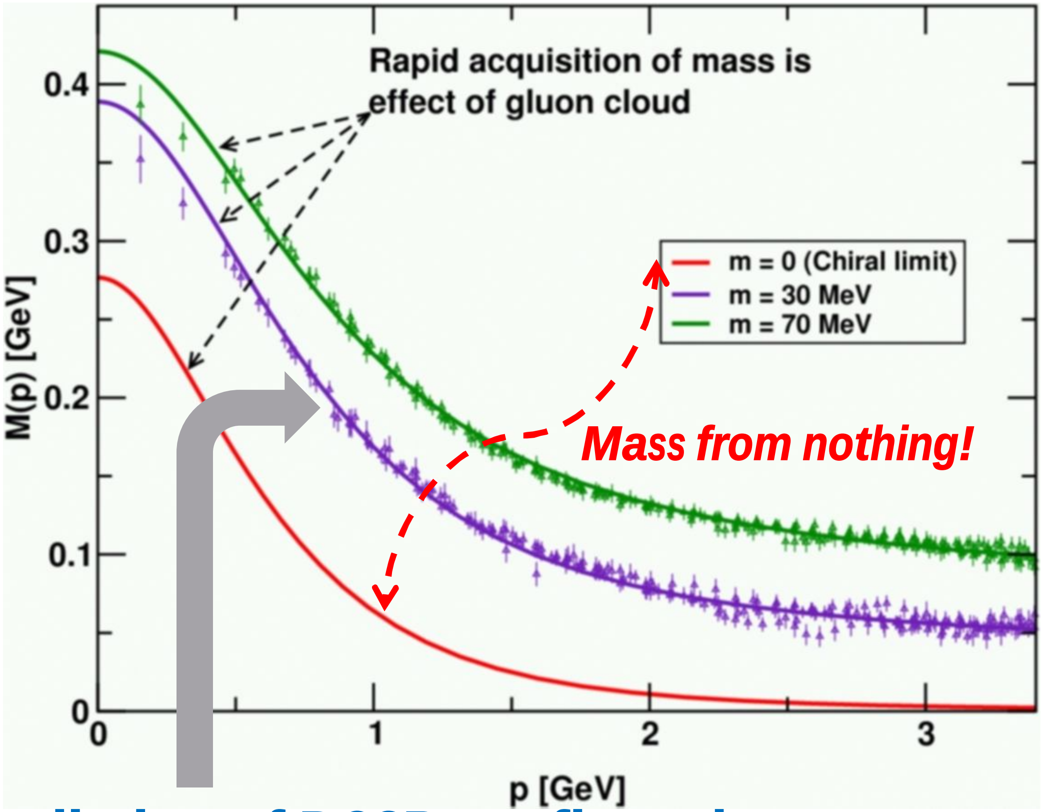
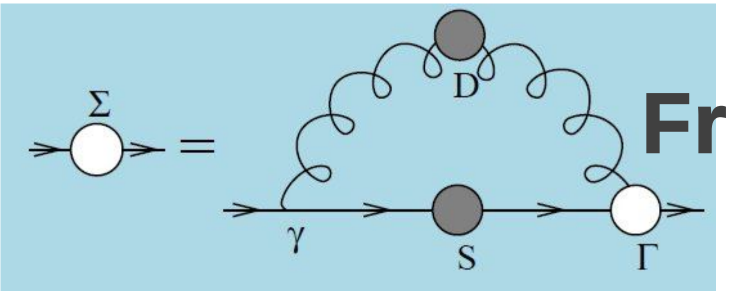


DSE prediction of DCSB confirmed



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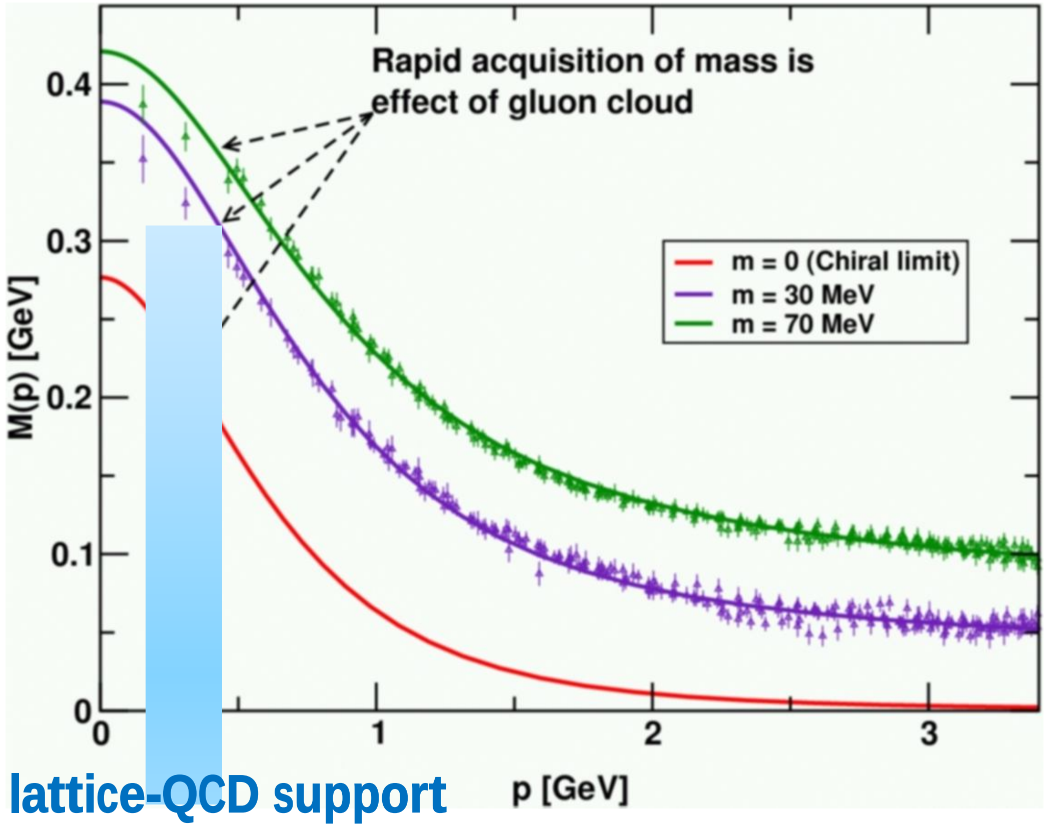
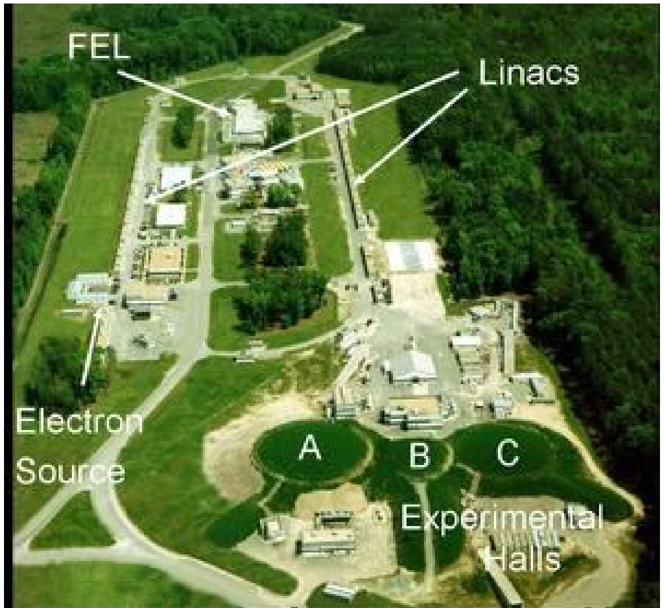


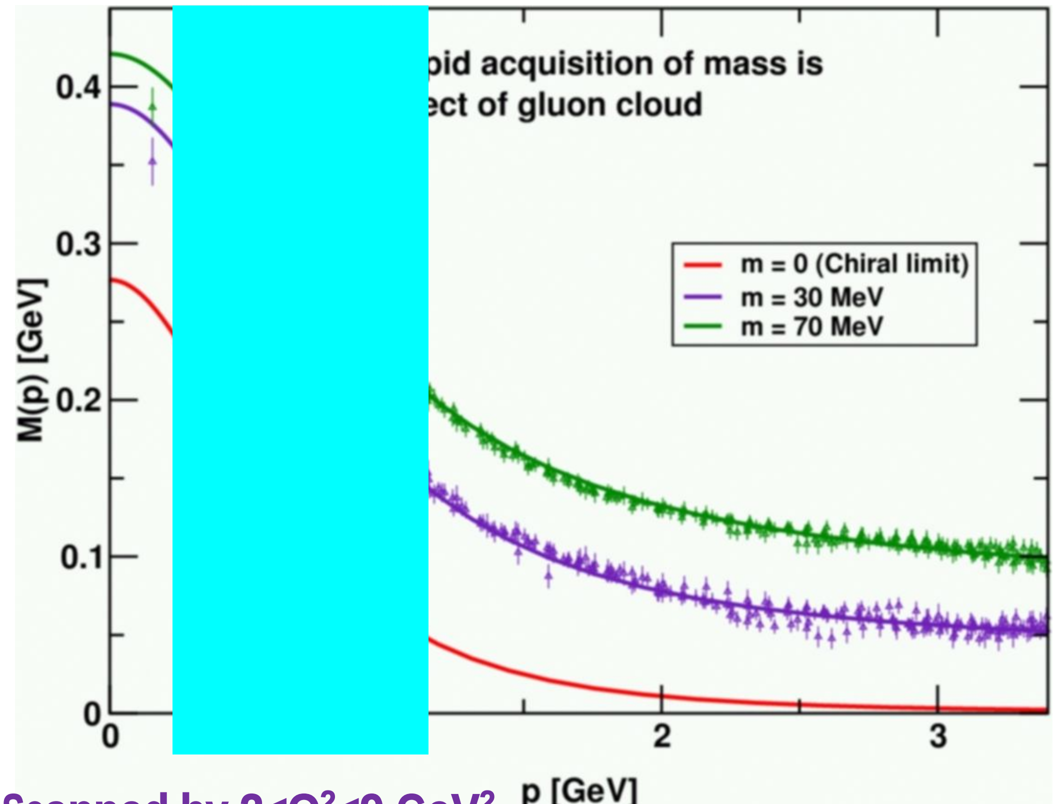
Hint of lattice-QCD support for DSE prediction of violation of reflection positivity



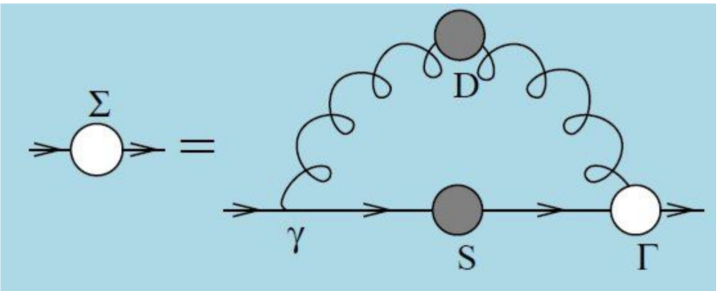
12GeV
The Future of JLab

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Numerical simulations of lattice QCD **(data, at two different bare masses)** **have confirmed model predictions** **(solid curves)** **that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates**. **In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.**



Jlab 12GeV: Scanned by $2<Q^2<9$ GeV^2 elastic & transition form factors.



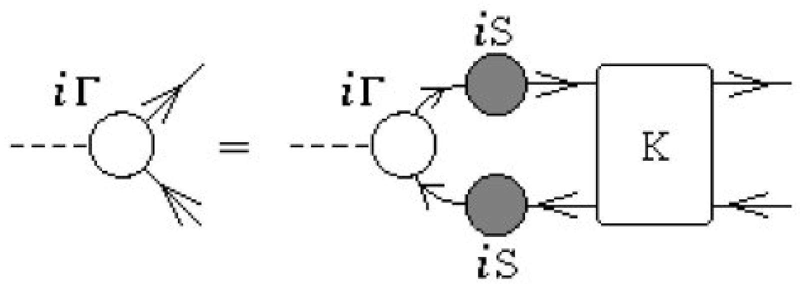
Gap Equation General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p)$$

- ***$D_{\mu\nu}(k)$ – dressed-gluon propagator***
- ***$\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex***
- **Suppose one has in hand – from anywhere – the exact form of the dressed-quark-gluon vertex**

What is the associated symmetry-preserving Bethe-Salpeter kernel?!



Bethe-Salpeter Equation
Bound-State DSE

$$\left[\Gamma^j_\pi(k;P)\right]_{tu} = \int_q^\Lambda \left[S(q+P/2)\Gamma^j_\pi(q;P)S(q-P/2)\right]_{sr} K^{rs}_{tu}(q,k;P)$$

- ***K(q,k;P) – fully amputated, two-particle irreducible, quark-antiquark scattering kernel***
- **Textbook material.**
- **Compact. Visually appealing. Correct**

Blocked progress for more than 60 years.

Bethe-Salpeter Equation General Form

Lei Chang and C.D. Roberts

0903.5461 [nucl-th]
Phys. Rev. Lett. 103 (2009) 081601

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu$$

$$- \boxed{\int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-)}$$

$$+ \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

- **Equivalent exact bound-state equation *but* in this form**

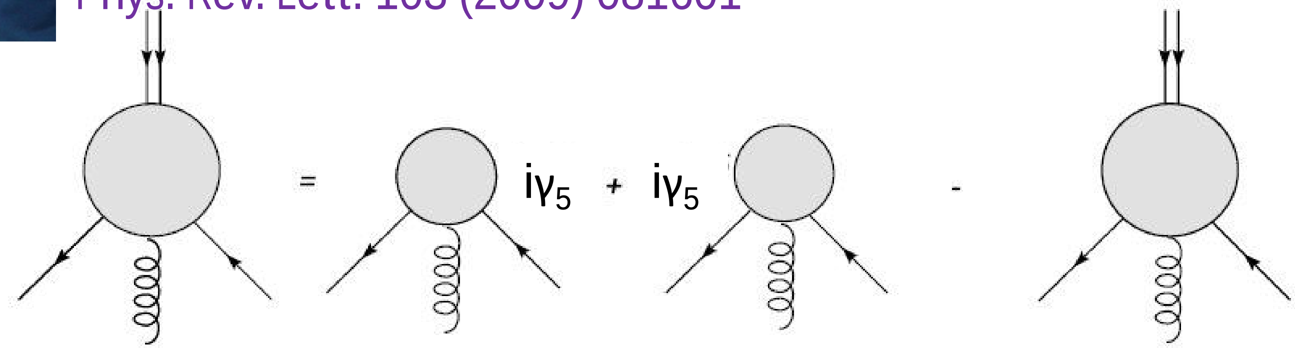
  K(q,k;P) → Λ(q,k;P)

  which is *completely determined by dressed-quark self-energy*

- **Enables derivation of a Ward-Takahashi identity for Λ(q,k;P)**

Ward-Takahashi Identity Bethe-Salpeter Kernel

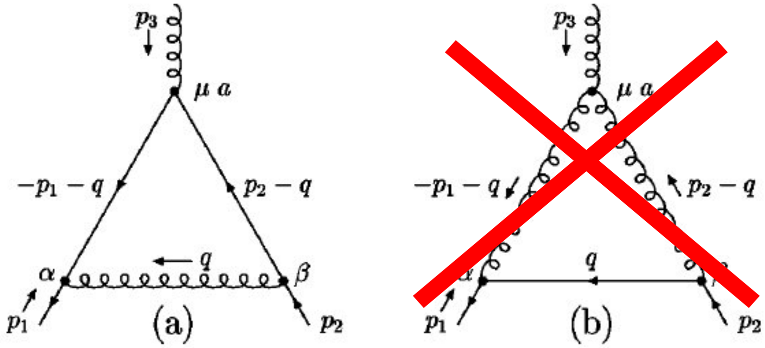
Lei Chang and C.D. Roberts
0903.5461 [nucl-th]
Phys. Rev. Lett. 103 (2009) 081601



$$
\begin{aligned}
P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) \;&=\; \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) \\
&\quad - i[m_f(\zeta)+m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),
\end{aligned}
$$

- **Now, for first time, it's possible to formulate an *Ansatz* for Bethe-Salpeter kernel given *any form* for the dressed-quark-gluon vertex by using this identity**
- **This enables the identification and elucidation of a wide range of novel consequences of DCSB**

Dressed-quark anomalous magnetic moments



- **Schwinger's result for QED:** $\frac{q}{2m} \to \left(1+\frac{\alpha}{2\pi}\right)\frac{q}{2m}$
- **pQCD: two diagrams**
  - **(a) is QED-like**
  - **(b) is only possible in QCD – involves 3-gluon vertex**
- **Analyse (a) and (b)**
  - **(b) vanishes identically: the 3-gluon vertex does *not* contribute to a quark's anomalous chromomag. moment at leading-order**
  - **(a) Produces a finite result: " – ⅙ $\alpha_s/2\pi$ "**

    ~ (– ⅙) QED-result
- **But, in QED and QCD, the *anomalous chromo- and electro-magnetic moments vanish identically in the chiral limit*!**

Dressed-quark anomalous magnetic moments

$$\int d^4x \, \tfrac{1}{2} q \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

- **Interaction term that describes magnetic-moment coupling to gauge field**
  - **Straightforward to show that it mixes left ↔ right**
  - **Thus, explicitly violates chiral symmetry**
- **Follows that in fermion's e.m. current**

  $\gamma_\mu F_1$ does cannot mix with $\sigma_{\mu\nu} q_\nu F_2$

  No Gordon Identity
  - **Hence *massless fermions cannot not possess a measurable chromo- or electro-magnetic moment***
- **But what if the chiral symmetry is dynamically broken, strongly, as it is in QCD?**

L. Chang, Y. –X. Liu and C.D. Roberts
arXiv:1009.3458 [nucl-th]
Phys. Rev. Lett. **106** (2011) 072001

Dressed-quark anomalous magnetic moments

- DCSB → **Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:**

Ball-Chiu term
•Vanishes if no DCSB
•Appearance driven by STI

$$\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$$

$$\Delta_F(p,q) = \frac{F(p^2)-F(q^2)}{p^2-q^2}$$

Anom. chrom. mag. mom. contribution to vertex
•Similar properties to BC term
•Strength commensurate with lattice-QCD
Skullerud, Bowman, Kizilersu *et al.* hep-ph/0303176

$$\Gamma^5_\mu(p,q) = \eta\, \sigma_{\mu\nu}(p-q)_\nu \Delta_B(p,q)$$

$$\Gamma^4_\mu(p,q) = [\ell^{\rm T}_\mu \gamma\cdot k + i\gamma^{\rm T}_\mu \sigma_{\nu\rho}\ell_\nu k_\rho]\tau_4(p,q)$$

$$\tau_4(p,q) = \mathcal{F}(z)\left[\frac{1-2\eta}{M_E}\Delta_B(p^2,q^2) - \Delta_A(p^2,q^2)\right]$$

$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2 + p_f^2 - 2M_E^2)/\Lambda_{\mathcal{F}}^2$, $\Lambda_{\mathcal{F}} = 1\,\text{GeV}$,
Simplifies numerical analysis;

$M_E = \{s|s>0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

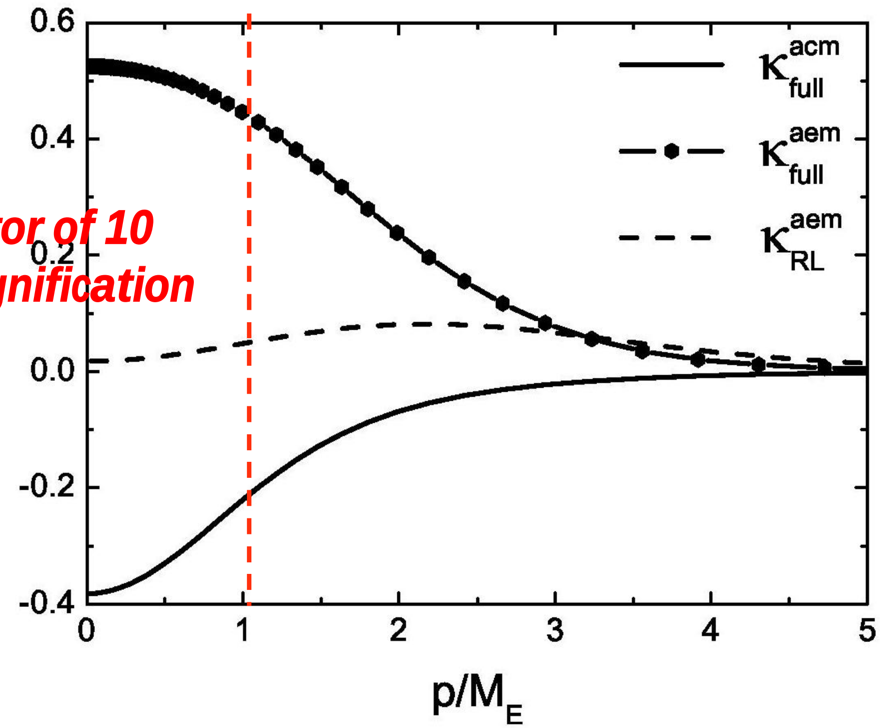
Role and importance is Novel discovery
•Essential to recover pQCD
•Constructive interference with Γ^5

L. Chang, Y. –X. Liu and C.D. Roberts
arXiv:1009.3458 [nucl-th]
Phys. Rev. Lett. **106** (2011) 072001

Dressed-quark anomalous magnetic moments

- Formulated and solved general Bethe-Salpeter equation
- Obtained dressed electromagnetic vertex
- Confined quarks don't have a mass-shell
  - o Can't unambiguously define magnetic moments
  - o But can define *magnetic moment distribution*
- AEM is opposite in sign but of roughly equal magnitude as ACM
  - o Potentially important for elastic & transition form factors, etc.
  - o Muon *g-2* – via Box diagram?

Factor of 10 magnification



	M^E	κ^{ACM}	κ^{AEM}
Full vertex	0.44	-0.22	0.45
Rainbow-ladder	0.35	0	0.048

Dressed Vertex
& Meson Spectrum

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885		
ρ	770	644	764		
Mass splitting	455	115	121		

- Splitting known experimentally for more than 35 years
- Hitherto, no explanation
- Systematic symmetry-preserving, Poincaré-covariant DSE truncation scheme of nucl-th/9602012.
  - Never better than ~ ¼ of splitting
- Constructing kernel skeleton-diagram-by-diagram, DCSB cannot be faithfully expressed: ***Full impact of $M(p^2)$ cannot be realised!***

Dressed Vertex & Meson Spectrum

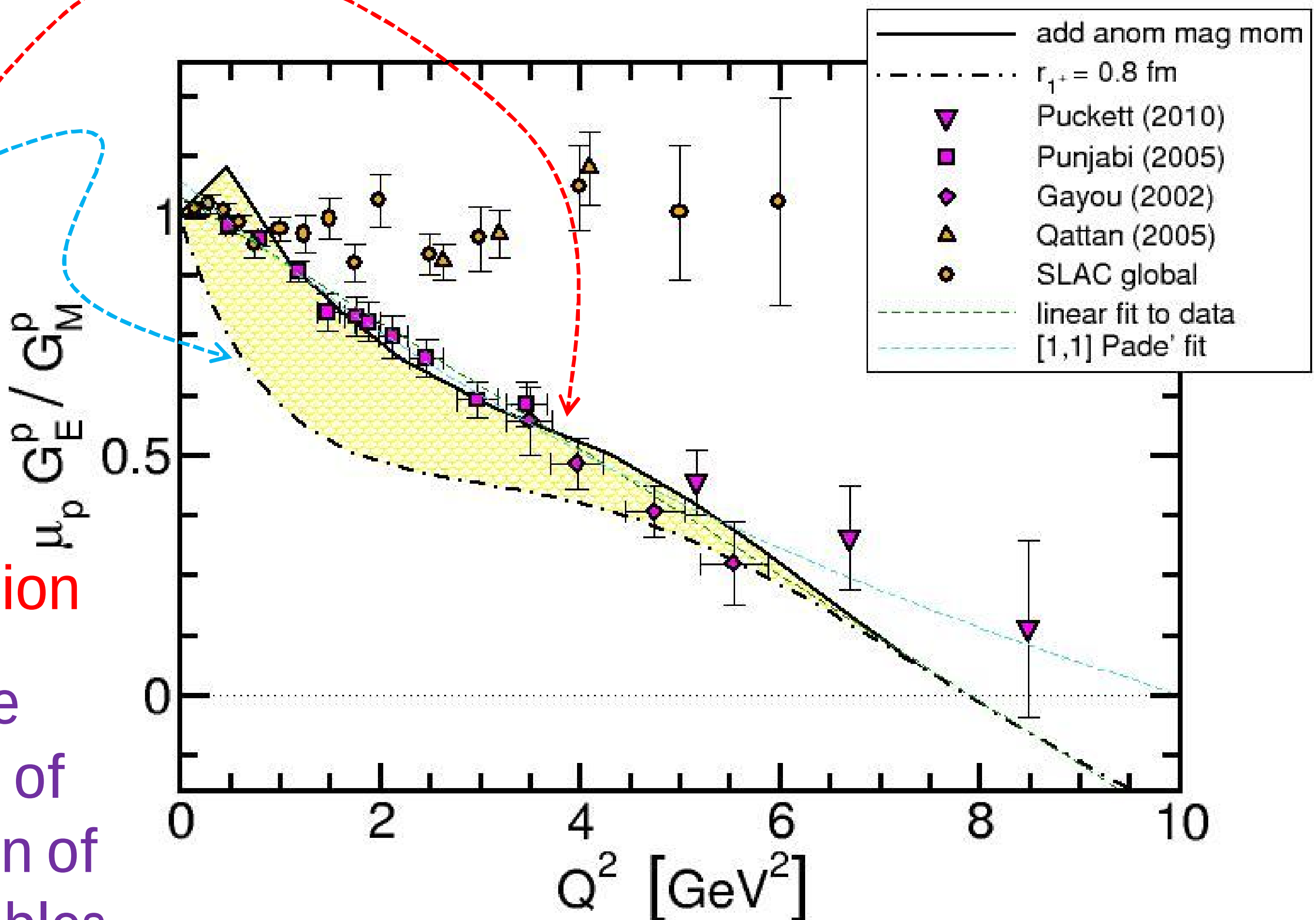
	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885	1066	1230
ρ	770	644	764	924	745
Mass splitting	455	115	121	142	485

- Fully consistent treatment of Ball-Chiu vertex
  - Retain λ_3 – term but ignore Γ^4 & Γ^5
  - Some effects of DCSB built into vertex & Bethe-Salpeter kernel
    - Big impact on σ – π complex
    - But, clearly, not the complete answer.
- Fully-consistent treatment of complete vertex *Ansatz*
- ***Promise of 1st reliable prediction of light-quark hadron spectrum***

I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]
I.C. Cloët, C.D. Roberts, *et al.*
In progress

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

- DSE result Dec 08
- DSE result
  – including the anomalous magnetic moment distribution
- Highlights again the critical importance of DCSB in explanation of real-world observables.



Pseudoscalar meson radial excitations
A. Höll, A. Krassnigg & C.D. Roberts,
Phys.Rev. C**70** (2004) 042203(R), nucl-th/0406030

Radial Excitations

- Goldstone modes are the only pseudoscalar mesons to possess a nonzero leptonic decay, f_π ,constant in the chiral limit when chiral symmetry is dynamically broken.
- The decay constants of their radial excitations vanish.
  - In quantum mechanics, decay constants are suppressed by a factor of roughly ⅓, but only a symmetry can ensure that something vanishes.
- Goldstone's Theorem for non-ground-state pseudoscalars
- These features and aspects of their impact on the meson spectrum were illustrated using a manifestly covariant and symmetry-preserving model of the kernels in the gap and Bethe-Salpeter equations.

A Chiral Lagrangian for excited pions
M.K. Volkov & C. Weiss
Phys. Rev. D**56** (1997) 221, hep-ph/9608347

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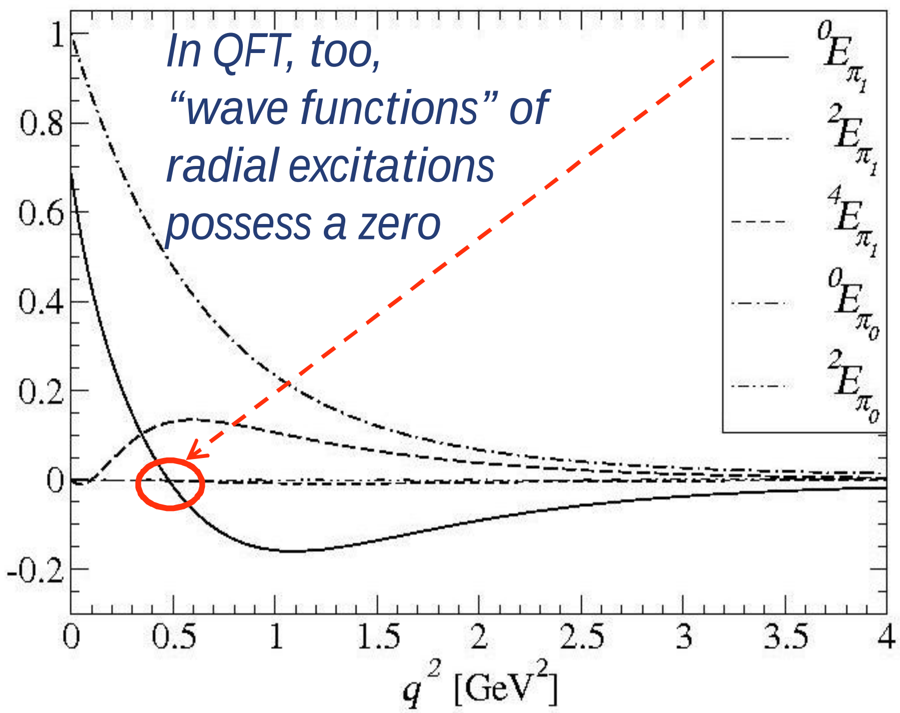

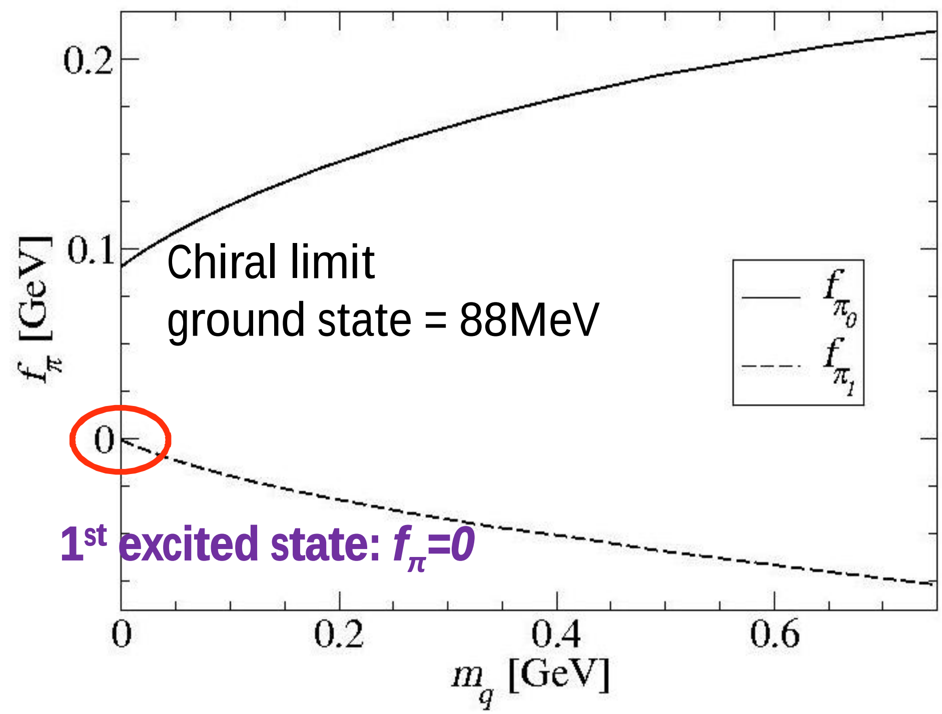

Pseudoscalar meson radial excitations
A. Höll, A. Krassnigg & C.D. Roberts,
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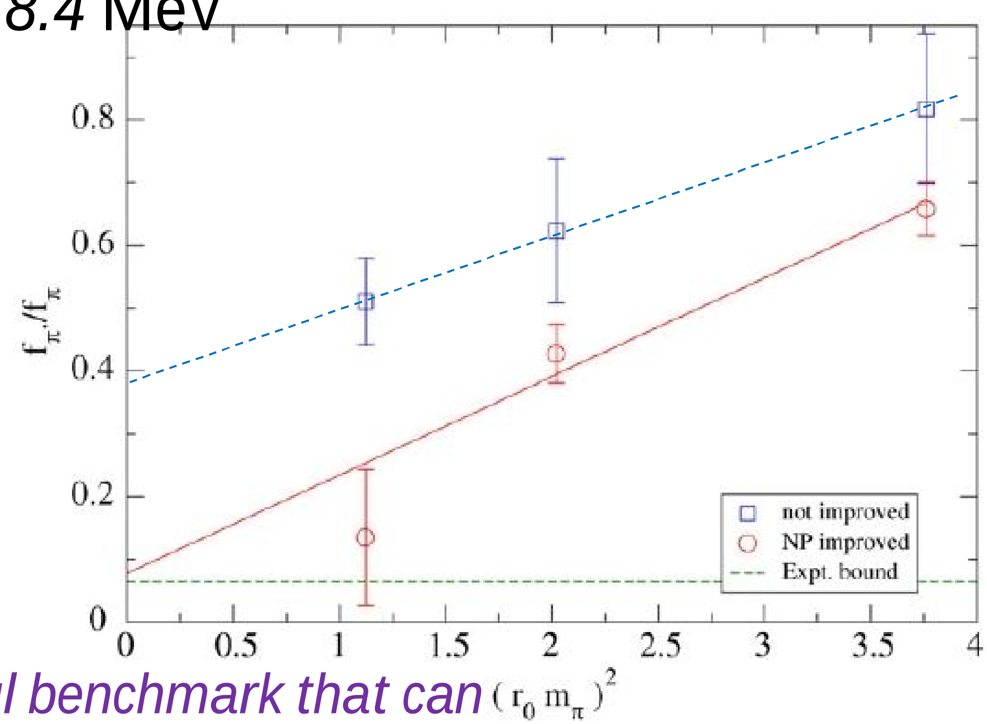
The Decay constant of the first excited pion from lattice QCD.
UKQCD Collaboration (C. McNeile, C. Michael *et al.*)
Phys. Lett. B**642** (2006) 244, hep-lat/0607032

Radial Excitations & Lattice-QCD

- When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"
- CLEO: $\tau \to \pi(1300)\, \nu_\tau$, $f_{\pi(1300)} < 8.4$ MeV
  Diehl & Hiller hep-ph/0105194
- Lattice-QCD check: $16^3 \times 32$,
  $a \sim 0.1$fm, 2-flavour, unquenched
  $f_{\pi(1300)} / f_{\pi(140)} = 0.078\ (93)$
- Full ALPHA formulation required to see suppression, because PCAC relation is at heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)



- *The suppression of $f_{\pi(1300)}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

R.T. Cahill *et al*.,
Austral. J. Phys. 42 (1989) 129-145

DSEs and Baryons

- *Dynamical chiral symmetry breaking* (DCSB)

  – has enormous impact on meson properties.

  - *Must be included in description and prediction of baryon properties*.

- *DCSB* is essentially a quantum field theoretical effect.

  In quantum field theory

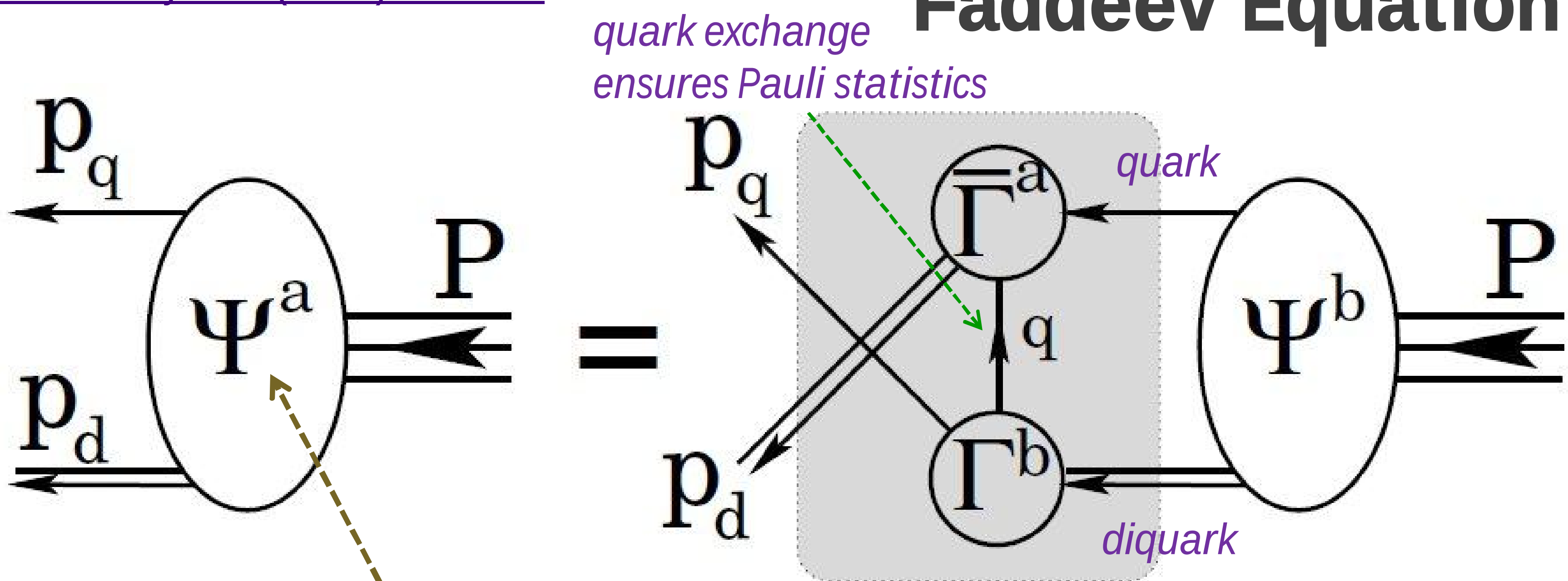
  - Meson appears as pole in four-point quark-antiquark Green function → Bethe-Salpeter Equation
  - *Nucleon appears as a pole in a six-point quark Green function → Faddeev Equation*.

- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates *nonpointlike* quark-quark (diquark) correlations in the colour-antitriplet channel

$r_{qq} \approx r_{\pi}$

R.T. Cahill *et al*.,
Austral. J. Phys. 42 (1989) 129-145

Faddeev Equation

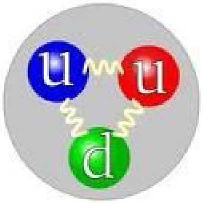


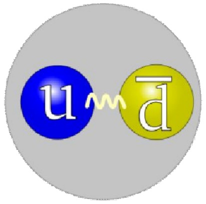
- Linear, Homogeneous Matrix equation
  - Yields *wave function (Poincaré Covariant Faddeev* Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . .
  - Both have "*correct*" parity and "*right*" masses
  - In Nucleon's Rest Frame Amplitude has

    s−, p− & d−wave correlations

Unification of Meson & Baryon Spectra

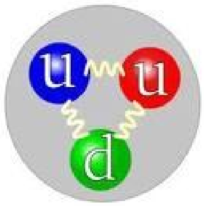
- Correlate the masses of meson and baryon ground- and excited-states within a *single, symmetry-preserving framework*
  - Symmetry-preserving means:
    Poincaré-covariant & satisfy relevant Ward-Takahashi identities
- Constituent-quark model has hitherto been the most widely applied spectroscopic tool; and whilst its weaknesses are emphasized by critics and acknowledged by proponents, it is of continuing value because there is nothing better that is yet providing a bigger picture.
- Nevertheless,
  - no connection with quantum field theory & certainly not with QCD
  - not symmetry-preserving & therefore cannot veraciously connect meson and baryon properties

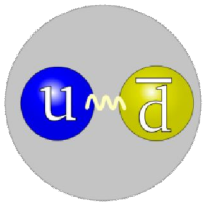




Unification of Meson & Baryon Spectra

- Dyson-Schwinger Equations have been applied extensively to the spectrum and interactions of mesons with masses less than 1 GeV
- On this domain the *rainbow-ladder* approximation, which is the leading-order in a systematic & symmetry-preserving truncation scheme – nucl-th/9602012, is an accurate, well-understood tool: e.g.,
  - Prediction of elastic pion and kaon form factors: nucl-th/0005015
  - Anomalous neutral pion processes – $\gamma\pi\gamma$ & BaBar anomaly: 1009.0067 [nucl-th]
  - Pion and kaon valence-quark distribution functions: 1102.2448 [nucl-th]
  - Unification of these and other observables – ππ scattering: hep-ph/0112015
- It can readily be extended to explain properties of the light neutral pseudoscalar mesons: 0708.1118 [nucl-th]



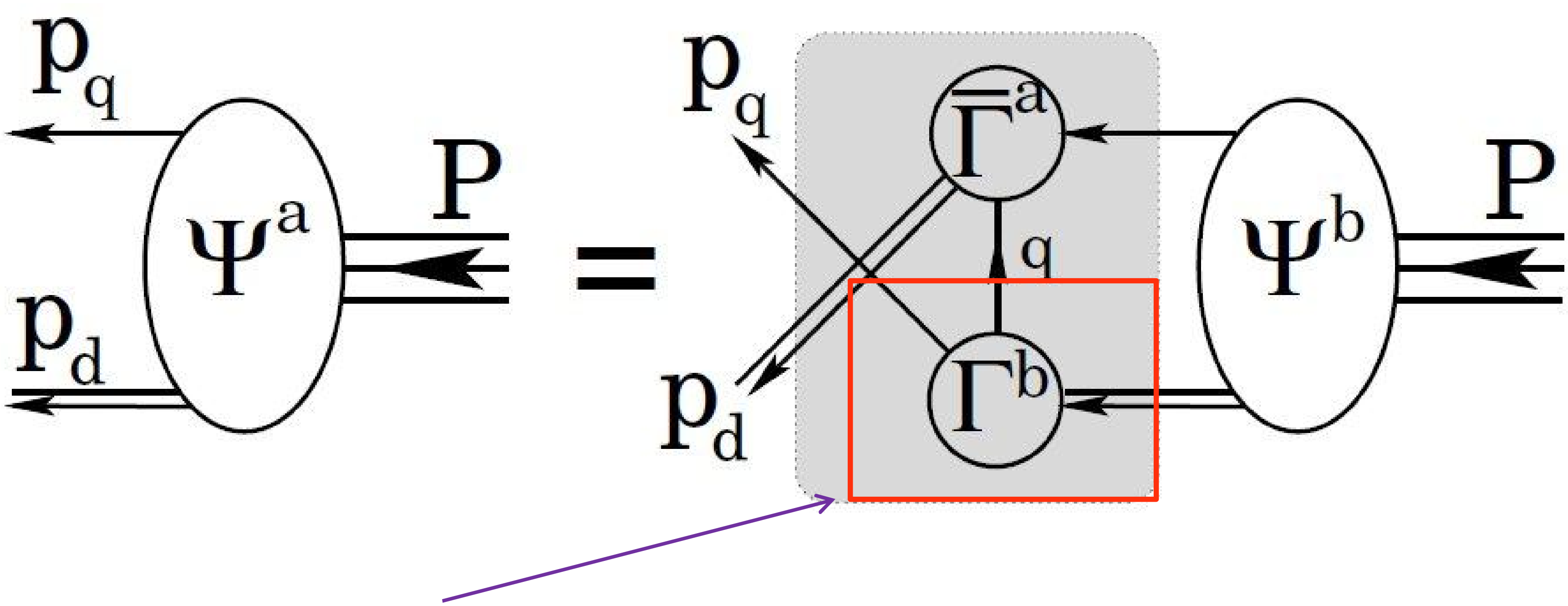


Unification of Meson & Baryon Spectra

- Some people have produced a spectrum of mesons with masses above 1GeV – but results have always been poor
  - For the bulk of such studies since 2004, this was a case of "*Doing what can be done, not what needs to be done*."
- Now understood why rainbow-ladder is not good for states with material angular momentum
  - know which channels are affected – scalar and axial-vector;
  - and the changes to expect in these channels
- Task – Improve rainbow-ladder for mesons

  & build this knowledge into Faddeev equation for baryons, because formulation of Faddeev equation rests upon knowledge of quark-quark scattering matrix

Faddeev Equation



quark-quark scattering matrix
- pole-approximation used to arrive at Faddeev-equation

Calculation of diquark masses in QCD
R.T. Cahill, C.D. Roberts and J. Praschifka
Phys.Rev. D**36** (1987) 2804

Diquarks

- Rainbow-ladder gap and Bethe-Salpeter equations

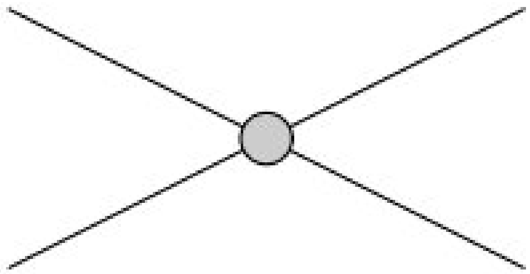
$$S(p)^{-1} = i\gamma\cdot p + m + \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2}\gamma_\mu S(q) \frac{\lambda^a}{2}\gamma_\nu(q,p),$$

$$\Gamma(k;P) = -\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2}\gamma_\mu S(q+P)\Gamma(q;P)S(q)\frac{\lambda^a}{2}\gamma_\nu\,.$$

- In this truncation, colour-antitriplet quark-quark correlations (diquarks) are described by a very similar homogeneous Bethe-Salpeter equation

$$\Gamma_{qq}(k;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2}\gamma_\mu S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\frac{\lambda^a}{2}\gamma_\nu$$

- Only difference is factor of ½
- Hence, an interaction that describes mesons also generates diquark correlations in the colour-antitriplet channel



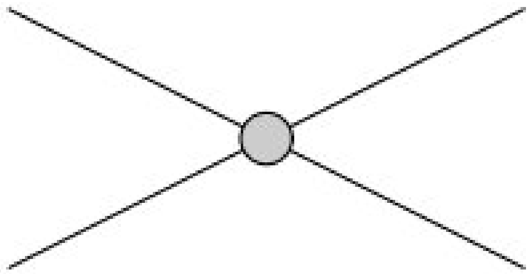
Interaction Kernel

- Vector-vector contact interaction

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \frac{1}{m_G^2}$$

m_G is a gluon mass-scale – dynamically generated in QCD

- Gap equation: $$M = m + \frac{M}{3\pi^2 m_G^2} \int_0^\infty ds\, s \frac{1}{s+M^2}$$
- DCSB: $M \neq 0$ is possible so long as $m_G < m_G^{critical}$
- Studies of π & ρ static properties and π form factor establish that contact-interaction results are not realistically distinguishable from those of renormalisation-group-improved one-gluon exchange for $Q^2 < M^2$

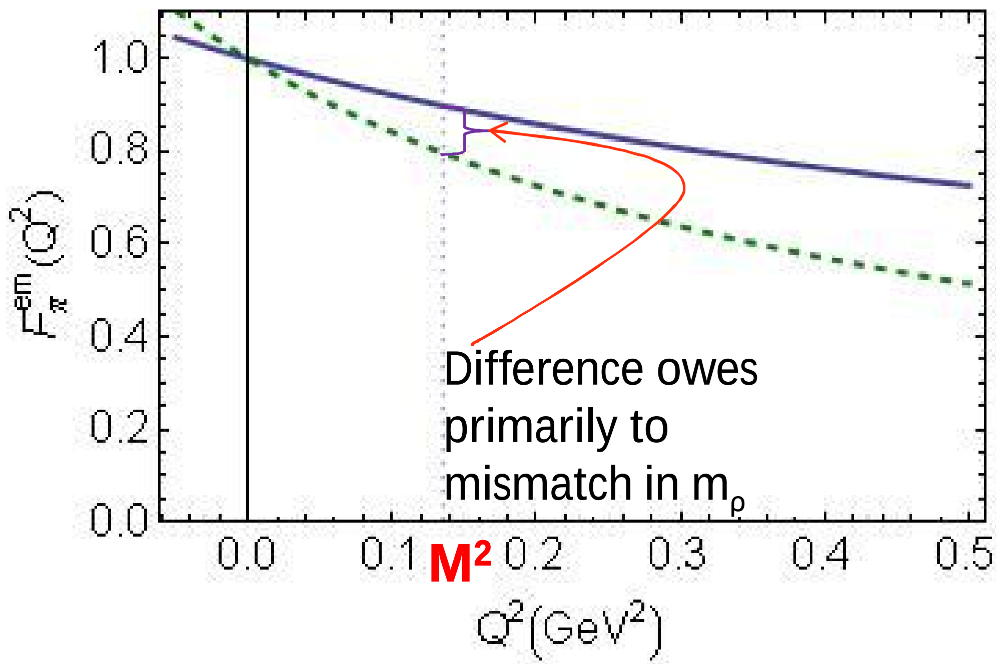


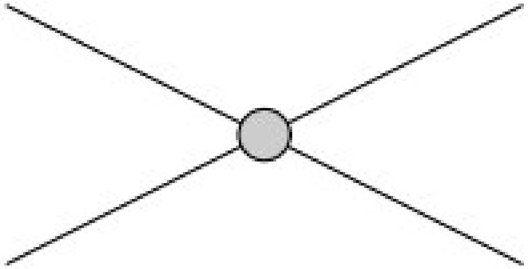
Interaction Kernel

- Studies of π & ρ static properties and π form factor establish that contact-interaction results are not realistically distinguishable from those of renormalisation-group-improved one-gluon exchange for $Q^2<M^2$

	contact interaction	QCD 1-loop RGI gluon
M	0.37	0.34
κ_π	0.24	0.24
m_π	0.14	0.14
m_ρ	0.93	0.74
f_π	0.10	0.093
f_ρ	0.13	0.15

cf. expt.
rms rel.err.=13%





Interaction Kernel - Regularisation Scheme

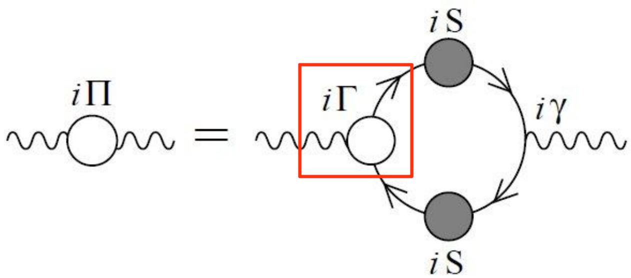
- Contact interaction is not renormalisable
- Must therefore introduce regularisation scheme
  - Use confining proper-time definition

D. Ebert, T. Feldmann and H. Reinhardt, Phys. Lett. B 388 (1996) 154.
No pole in propagator
– DSE realisation of confinement

$$\frac{1}{s+M^2} = \int_0^\infty d\tau\, \mathrm{e}^{-\tau(s+M^2)} \rightarrow \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\, \mathrm{e}^{-\tau(s+M^2)} = \frac{\mathrm{e}^{-(s+M^2)\tau_{\rm uv}^2} - e^{-(s+M^2)\tau_{\rm ir}^2}}{s+M^2}$$

- $\Lambda_{ir} = 0.24$GeV, $\tau_{ir} = 1/\Lambda_{ir} = 0.8$fm
  a confinement radius, which is not varied
- Two parameters:
  $m_G=0.13$GeV, $\Lambda_{uv}=0.91$GeV
  fitted to produce tabulated results

	contact interaction
M	0.37
κ_π	0.24
m_π	0.14
m_ρ	0.93
f_π	0.10
f_ρ	0.13



Regularisation & Symmetries

- In studies of hadron spectrum it's critical that an approach satisfy the vector and axial-vector Ward-Takahashi identities.
  - Without this it is impossible to preserve the pattern of chiral symmetry breaking in QCD & hence a veracious understanding of hadron mass splittings is not achievable.
- Contact interaction should & can be regularised appropriately
- Example: dressed-quark-photon vertex
  - Contact interaction plus rainbow-ladder entails general form

$$\Gamma_\mu(k;Q) = \gamma_\mu^T P_T(Q^2) + \gamma_\nu^L P_L(Q^2)$$

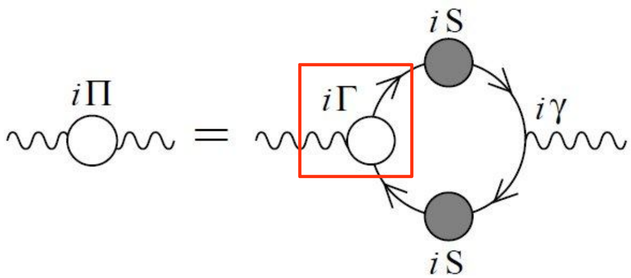
  - Vector Ward-Takahashi identity

$$Q_\mu i\Gamma_\mu(k;Q) = S^{-1}(k+Q/2) - S^{-1}(k-Q/2)$$

  - With symmetry-preserving regularisation of contact interaction, identity requires

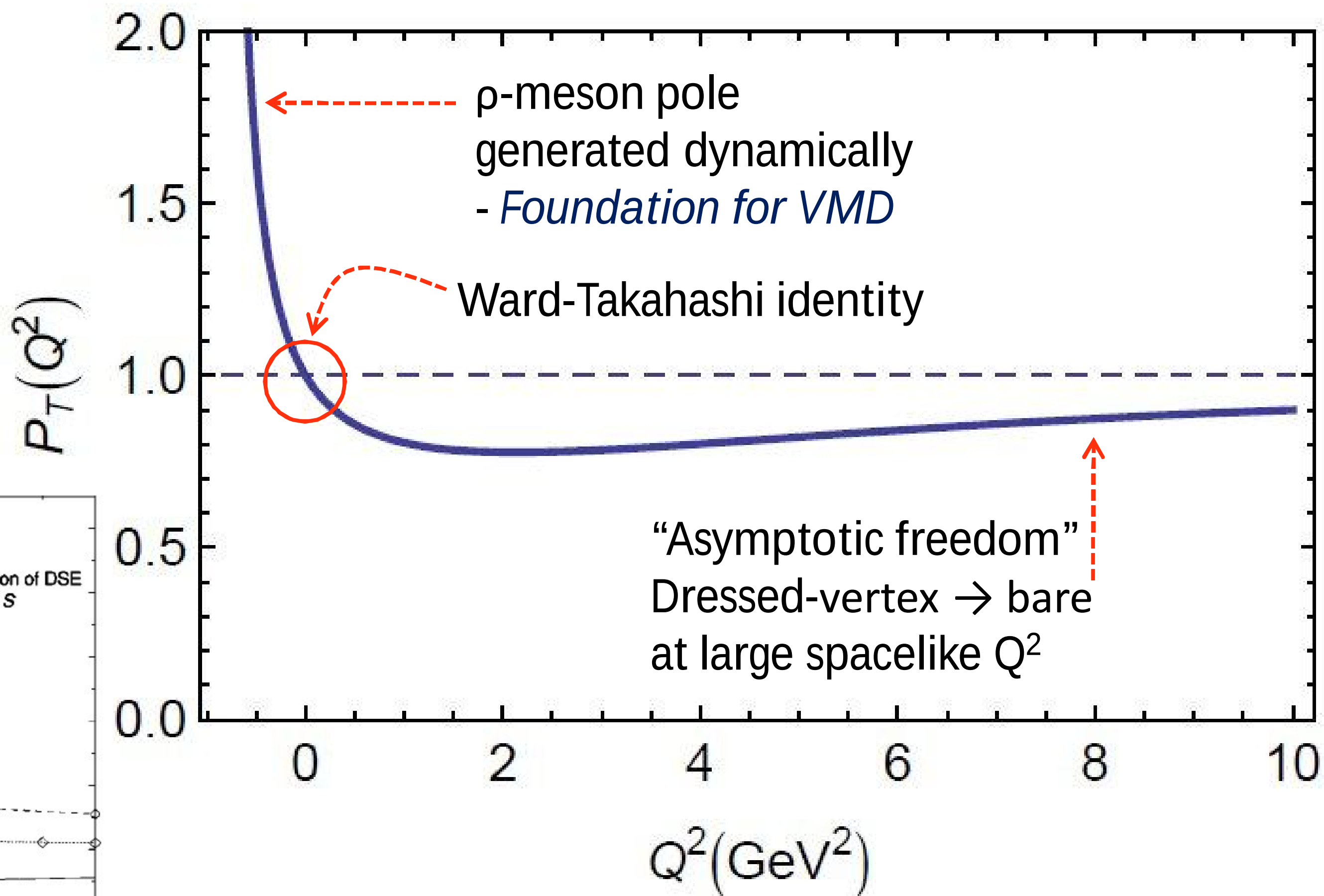
$$P_L(Q^2)=1 \ \& \ P_T(Q^2=0)=1$$

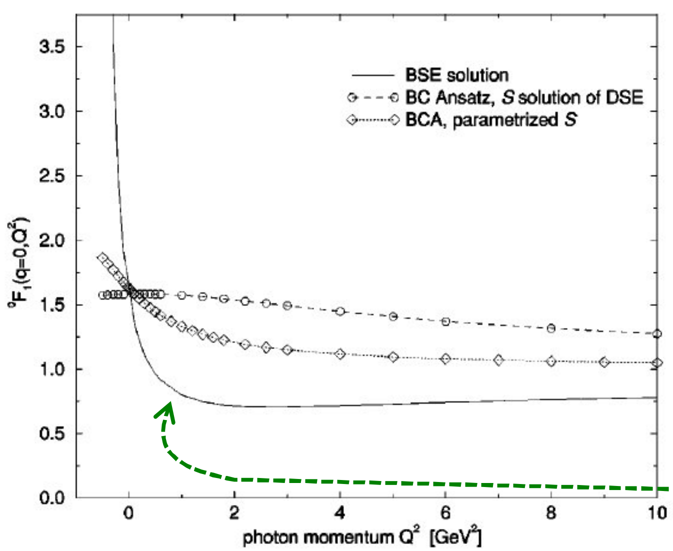
Interactions cannot generate an on-shell mass for the photon.



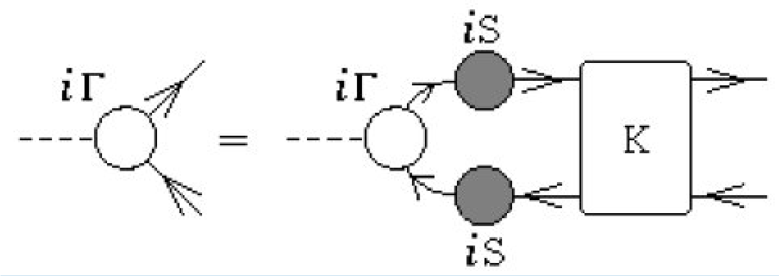
Regularisation & Symmetries

- Solved Bethe-Salpeter equation for dressed-quark photon vertex, regularised using symmetry-preserving scheme





RGI one-gluon exchange
Maris & Tandy prediction of $F_\pi(Q^2)$



Bethe-Salpeter Equations

- Ladder BSE for ρ-meson

$$1+K^{\rho}(-m_{1-}^2)=0\,,\ K^{\rho}(P^2)=\frac{1}{3\pi^2 m_G^2}\int_0^1 d\alpha\,\alpha(1-\alpha)P^2\,\overline{\mathcal{C}}_1^{iu}(\omega(M^2,\alpha,P^2))$$

$$\overline{C}_1^{iu}(\omega)=\Gamma(0,M^2 r_{uv}^2)-\Gamma(0,M^2 r_{ir}^2),\ C_1^{iu}(\omega)=\omega\overline{C}_1^{iu}(\omega)$$

$\omega(M^2,\alpha,P^2)= M^2+\alpha(1-\alpha)P^2$

  - *Contact interaction, properly regularised, provides a practical simplicity and physical transparency*

- Ladder BSE for a_1-meson

$$1+K^{a_1}(-m_{1+}^2)=0\,,\ K^{a_1}(P^2)=-\frac{1}{3\pi^2 m_G^2}\int_0^1 d\alpha\,\mathcal{C}_1^{\mathrm{iu}}(\omega(M^2,\alpha,P^2))$$

- All BSEs are one- or –two dimensional eigenvalue problems, eigenvalue is P^2= - (mass-bound-state)^2

Meson Spectrum -Ground-states

- Ground-state masses
  - Computed very often, always with same result

	m_π	m_ρ	m_σ	m_{a_1}
RL	0.14	0.93	0.74	1.08
experiment	0.14	0.78	0.4 – 1.2	1.24

But, we know how to fix that viz., DCSB – a beyond rainbow ladder
- increases scalar and axial-vector masses
- leaves π & ρ unchanged

- Namely, with rainbow-ladder truncation

$m_{a1} - m_\rho = 0.15$ GeV $\approx \frac{1}{3} \times 0.45_{\text{experiment}}$

	Experiment	Rainbow-ladder	One-loop corrected	Full vertex
a1	1230	759	885	1230
ρ	770	644	764	745
Mass splitting	455	115	121	485

Meson Spectrum -Ground-states

- Ground-state masses
  - Correct for omission of DCSB-induced spin-orbit repulsion

	m_π	m_ρ	m_σ	m_{a_1}
RL	0.14	0.93	0.74	1.08
experiment	0.14	0.78	0.4 – 1.2	1.24

$m_\sigma^{qq} \approx 1.2$ GeV is location of quark core of σ-resonance:

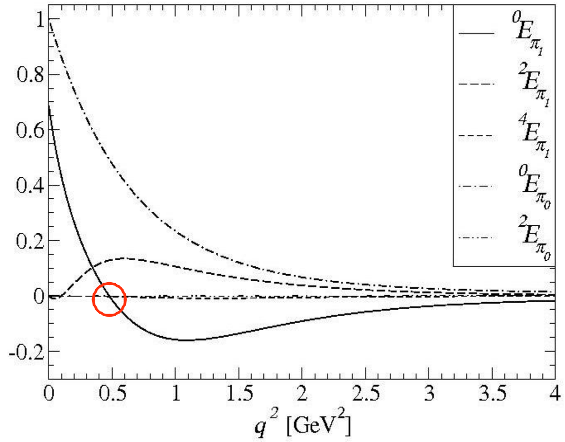
- Pelaez & Rios (2006)
- Ruiz de Elvira, Pelaez, Pennington & Wilson (2010)

First novel post-diction

- Leave *π*- & *ρ*-meson BSEs unchanged but introduce repulsion parameter in scalar and axial-vector channels; viz.,

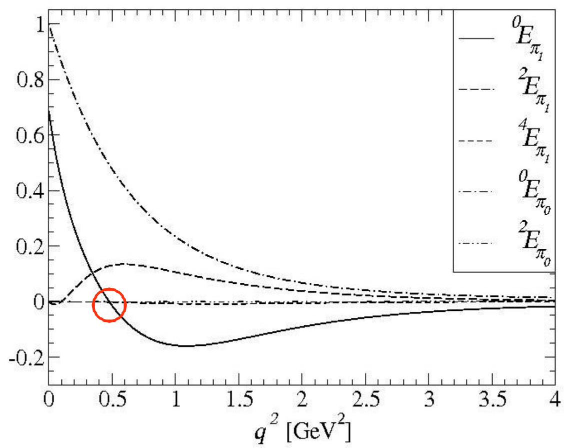
$$1 + K^{a1}(-m_{1+}^2) = 0\,,\ K^{a1}(P^2) = -\frac{\boxed{g_{SO}^2}}{3\pi^2 m_G^2}\int_0^1 d\alpha\, \mathcal{C}_1^{\rm iu}(\omega(M^2,\alpha,P^2))$$

- $g_{SO}=0.24$ fitted to produce $m_{a1} - m_\rho = 0.45_{\text{experiment}}$



Meson Spectrum - Radial Excitations

- As illustrated previously, radial excitations possess a single zero in the relative-momentum dependence of the leading Tchebychev-moment in their Bethe-Salpeter amplitude
- The existence of radial excitations is therefore very obvious evidence against the possibility that the interaction between quarks is momentum-independent:
  - *A bound-state amplitude that is independent of the relative momentum cannot exhibit a single zero*
- One may express this differently; viz.,
  - If the location of the zero is at k_0^2, then a momentum-independent interaction can only produce reliable results for phenomena that probe momentum scales $k^2 < k_0^2$.
  - In QCD, $k_0 \approx M$.



Meson Spectrum - Radial Excitations

- Nevertheless, there exists an established expedient ; viz.,
  - Insert a zero by hand into the Bethe-Salpeter kernels

(1-k^2/M^2)

$$1 + K^{\rho *}(-m_{1^-}^2) = 0\,,\; K^{\rho *}(P^2) = \frac{1}{3\pi^2 m_G^2}\int_0^1 d\alpha\, \alpha(1-\alpha) P^2\, \bar{\mathcal{C}}_1^{iu}(\omega(M^2,\alpha,P^2))$$

A Chiral Lagrangian for excited pions
M.K. Volkov & C. Weiss
Phys. Rev. D**56** (1997) 221, hep-ph/9608347

- Plainly, the presence of this zero has the effect of reducing the coupling in the BSE & hence it increases the bound-state's mass.
- Although this may not be as transparent with a more sophisticated interaction, a qualitatively equivalent mechanism is always responsible for the elevated values of the masses of radial excitations.
- Location of zero fixed at “natural” location – not a parameter

plus predicted diquark spectrum

Meson Spectrum Ground- & Excited-States

- Complete the table …

	m_π	m_ρ	m_σ	m_{a_1}
RL	0.14	0.93	0.74	1.08
RL $* g_{SO}^2$	0.14	0.93	1.29	1.38
experiment	0.14	0.78	0.4 – 1.2	1.24

- Error estimate for radial excitations:

  Shift location of zero by ±20%

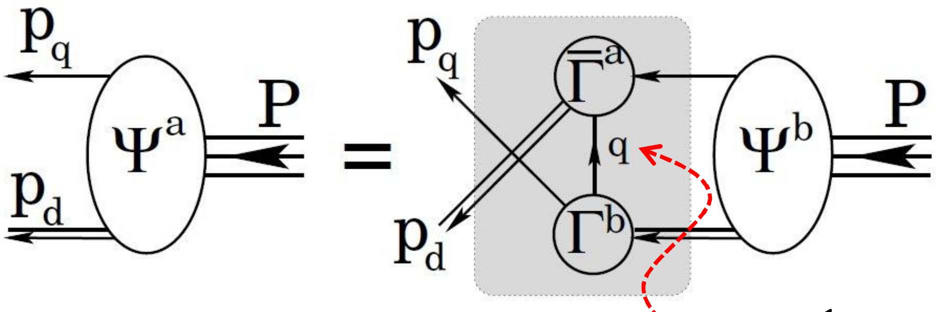
- *rms-relative-error/degree-of-freedom = 13%*

	$m_{qq_{0+}}$	$m_{qq_{1+}}$	$m_{qq_{0-}}$	$m_{qq_{1-}}$	$m_{qq^*_{0+}}$	$m_{qq^*_{1+}}$	$m_{qq^*_{0-}}$	$m_{qq^*_{1-}}$
RL	0.78	1.06	0.93	1.16	1.39 ± 0.06	1.32 ± 0.05	1.42 ± 0.05	1.33 ± 0.05
RL $* g_{SO}^2$	0.78	1.06	1.37	1.45	1.39 ± 0.06	1.32 ± 0.05	1.50 ± 0.03	1.52 ± 0.02

- *No parameters*
- *Realistic* DSE estimates: $m_{0+}=0.7\text{-}0.8$, $m_{1+}=0.9\text{-}1.0$
- Lattice-QCD estimate: $m_{0+}=0.78 \pm 0.15$, $m_{1+}\text{-}m_{0+}=0.14$

NO results for other *qq* quantum numbers, critical for excited states of N and Δ

Spectrum of Baryons

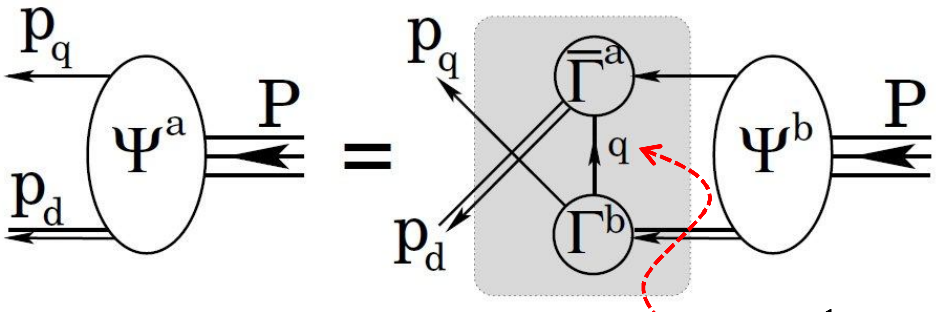


$$\frac{1}{i\gamma\cdot p + M} \to \frac{g_B^2}{M}$$

Variant of:
A. Buck, R. Alkofer & H. Reinhardt, Phys. Lett. ***B286*** *(1992) 29.*

- Static "approximation"
  - Implements analogue of contact interaction in Faddeev-equation
- In combination with contact-interaction diquark-correlations, generates Faddeev equation kernels which themselves are momentum-independent
- The merit of this truncation is the *dramatic simplifications* which it produces
- Used widely in hadron physics phenomenology; e.g., Bentz, Cloët, Thomas *et al*.

Spectrum of Baryons

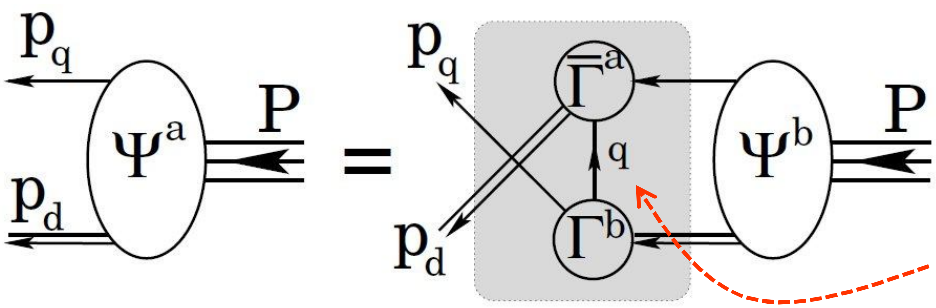


$$\frac{1}{i\gamma \cdot p + M} \to \frac{g_B^2}{M}$$

- Static "approximation"
  - Implements analogue of contact interaction in Faddeev-equation
- From the referee's report:

In these calculations one could argue that the [static truncation] *is the weakest* [approximation]*. From what I understand, it is not of relevance here since the aim is to understand the dynamics of the interactions between the* [different] *types of diquark correlations with the spectator quark and their different contributions to the baryon's masses … this study illustrates rather well what can be expected from more sophisticated models, whether within a Dyson-Schwinger or another approach. … I can recommend the publication of this paper without further changes.*

Spectrum of Baryons



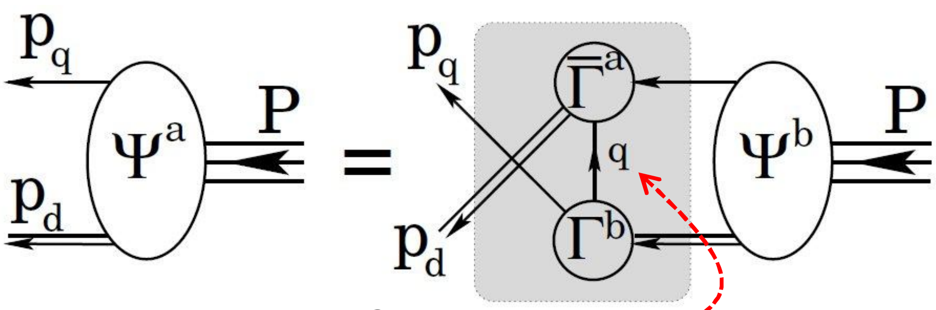
$$\frac{1}{i\gamma \cdot p + M} \rightarrow \frac{g_B^2}{M}$$

- Faddeev equation for Δ-resonance

$$1 = 8 \frac{g_\Delta^2}{M} \frac{E_{qq_{1+}}^2}{m_{qq_{1+}}^2} \int \frac{d^4\ell'}{(2\pi)^4} \int_0^1 d\alpha \, \frac{(m_{qq_{1+}}^2 + (1-\alpha)^2 m_\Delta^2)(\alpha m_\Delta + M)}{[\ell'^2 + \sigma_\Delta(\alpha, M, m_{qq_{1+}}, m_\Delta)]^2}$$

$$= \frac{g_\Delta^2}{M} \frac{E_{qq_{1+}}^2}{m_{qq_{1+}}^2} \frac{1}{2\pi^2} \int_0^1 d\alpha \, (m_{qq_{1+}}^2 + (1-\alpha)^2 m_\Delta^2)(\alpha m_\Delta + M) \overline{\mathcal{C}}_1^{\mathrm{iu}}(\sigma_\Delta(\alpha, M, m_{qq_{1+}}, m_\Delta))$$

- *One-dimensional eigenvalue problem, to which only axial-vector diquark contributes*
- *Nucleon has scalar & axial-vector diquarks. It is a three-dimensional eigenvalue problem*



$$\frac{1}{i\gamma\cdot p+M}\rightarrow\frac{g_B^2}{M}$$

Spectrum of Baryons “pion cloud”

- Pseudoscalar-meson loop-corrections to our truncated DSE kernels
  - may have a material impact on m_N and m_Δ separately but the contribution to each is approximately the same
  - so that the mass-difference is largely unaffected by such corrections: $(m_\Delta - m_N)_{\pi\text{-loops}} = 40$MeV

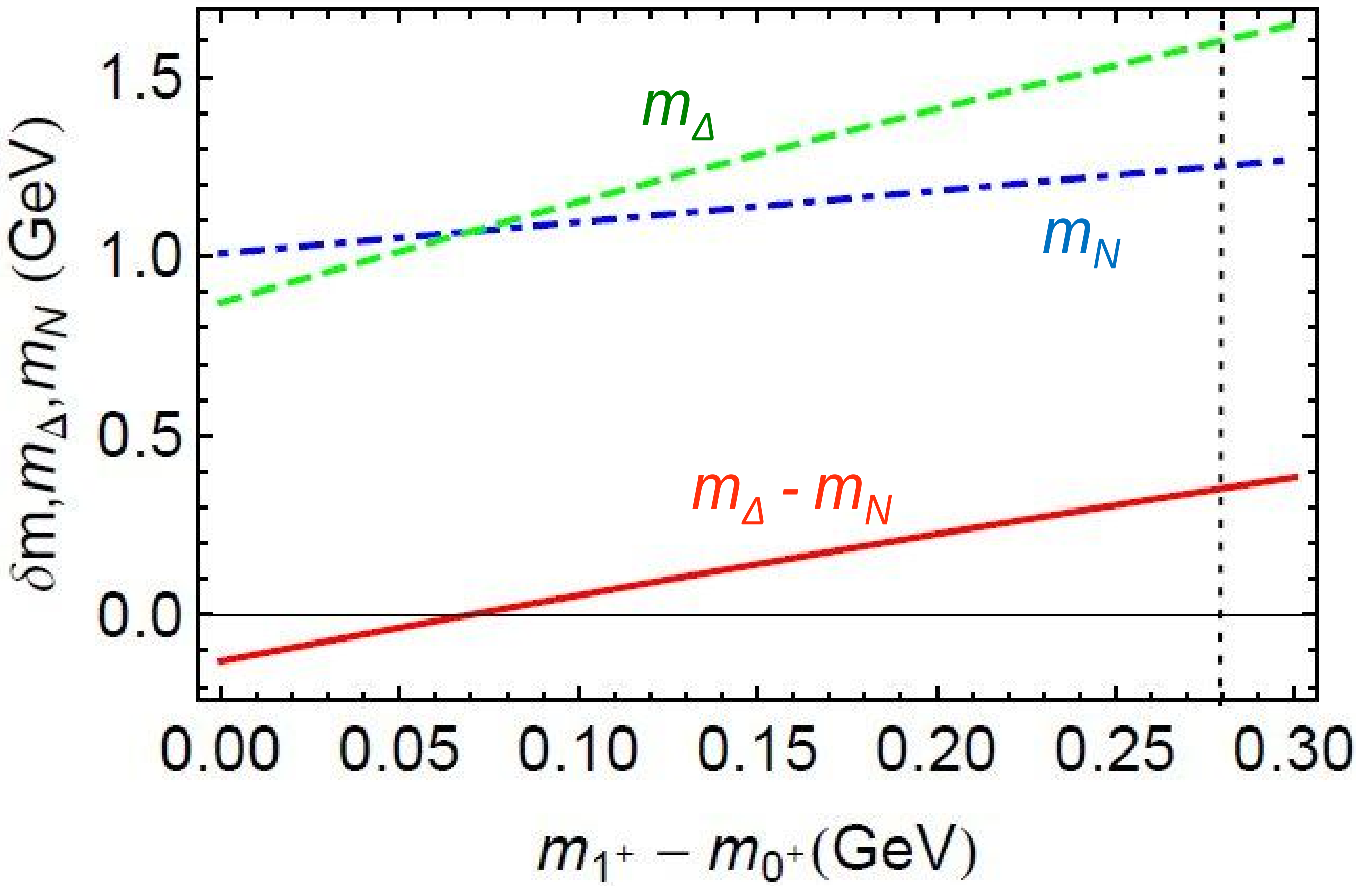
1. EBAC: “undressed Δ” has $m_\Delta = 1.39$GeV;
2. $(m_\Delta - m_N)_{qqq\text{-core}} = 250$MeV

achieved with $g_N=1.18$ & $g_\Delta=1.56$

All three spectrum parameters now fixed $(g_{SO}=0.24)$

Baryons & diquarks

- Provided numerous insights into baryon structure; e.g.,
  - *There is a causal connection between m_Δ - m_N & m_{1+}- m_{0+}*



Physical splitting grows rapidly with increasing diquark mass difference

Baryons & diquarks

- Provided numerous insights into baryon structure; e.g.,
  - $m_N \approx 3\,M$ & $m_\Delta \approx M + m_{1+}$

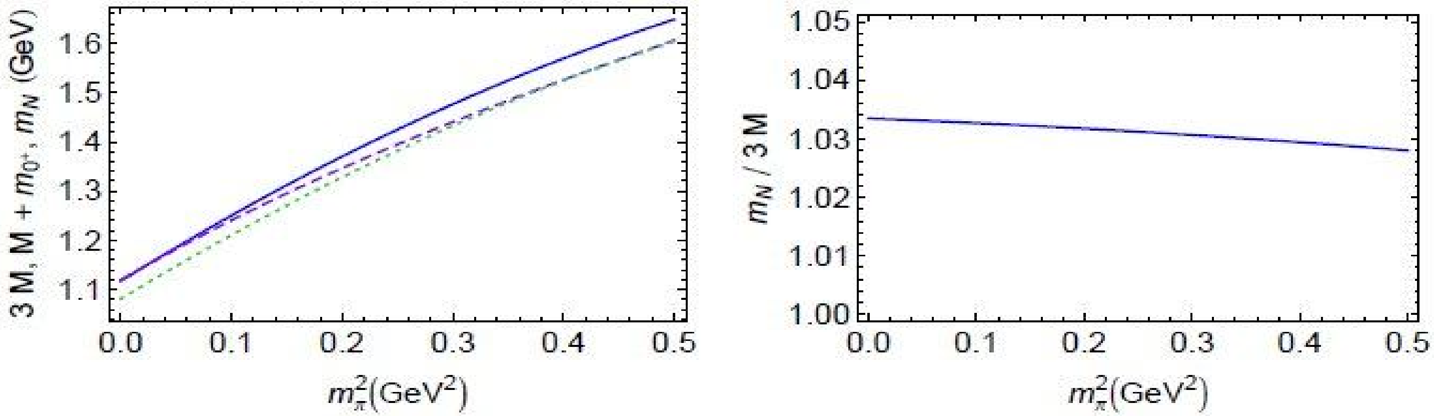


Fig. 3 *Left panel* – Evolution with current-quark mass of the: nucleon mass, m_N (*solid curve*); the sum $[M + m_{qq_{0^+}}]$ (*dashed curve*); and $3M$ (*dotted curve*). *Right panel* – Evolution with current-quark mass of the ratio $m_N/[3M]$, which varies by less-than 1% on the domain depicted.

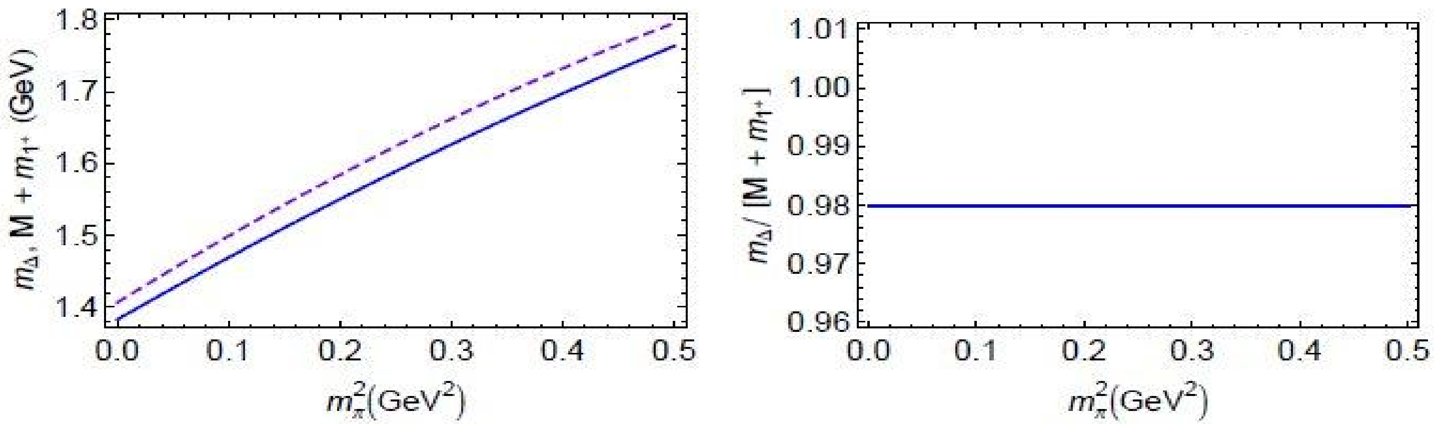


Fig. 4 *Left panel* – Evolution with current-quark mass of the: Δ mass, m_Δ (*solid curve*); and $[M + m_{qq_{1^+}}]$ (dashed curve). *Right panel* – Evolution with current-quark mass of the ratio $m_\Delta/[M + m_{qq_{1^+}}]$, which does not vary noticeably on the domain depicted.

Baryon Spectrum

Table 4 *Row-1*: Dressed-quark-core masses for nucleon and Δ, their first radial excitations (denoted by "$*$"), and the parity-partners of these states, computed with $g_N = 1.18$, $g_\Delta = 1.56$, and the parameter values in Eq. (25) and Table 1. The errors on the masses of the radial excitations indicate the effect of shifting the location of the zero according to Eq. (30). *Row-2*: Bare-masses inferred from a coupled-channels analysis at the Excited Baryon Analysis Center (EBAC) [65]. EBAC's method does not provide a bare nucleon mass. *Row-3*: Bare masses inferred from the coupled-channels analysis described in Ref. [67], which describes the Roper resonance as dynamically-generated. In both these rows, "…" indicates states not found in the analysis. A visual comparison of these results is presented in Fig. 7.

	m_N	m_{N^*}	$m_{N\frac{1}{2}^-}$	$m_{N^*\frac{1}{2}^-}$	m_Δ	m_{Δ^*}	$m_{\Delta\frac{3}{2}^-}$	$m_{\Delta^*\frac{3}{2}^-}$
PDG label	N	$N(1440)\,P_{11}$	$N(1535)\,S_{11}$	$N(1650)\,S_{11}$	$\Delta(1232)\,P_{33}$	$\Delta(1600)\,P_{33}$	$\Delta(1700)\,D_{33}$	$\Delta(1940)\,D_{33}$
This work	1.14	1.82 ± 0.07	2.22	2.29 ± 0.02	1.39	1.85 ± 0.05	2.25	2.33 ± 0.02
EBAC		1.76	1.80	1.88	1.39	…	1.98	…
Jülich	1.24	none	2.05	1.92	1.46	…	2.25	…

- Our predictions for baryon dressed-quark-core masses match the bare-masses determined by Jülich with a rms-relative-error of 10%.
  - *Notably, however, we find a quark-core to the Roper resonance, whereas within the Jülich coupled-channels model this structure in the P_{11} partial wave is unconnected with a bare three-quark state.*

Baryon Spectrum

Table 4 *Row-1*: Dressed-quark-core masses for nucleon and Δ, their first radial excitations (denoted by "$*$"), and the parity-partners of these states, computed with $g_N = 1.18$, $g_\Delta = 1.56$, and the parameter values in Eq. (25) and Table 1. The errors on the masses of the radial excitations indicate the effect of shifting the location of the zero according to Eq. (30). *Row-2*: Bare-masses inferred from a coupled-channels analysis at the Excited Baryon Analysis Center (EBAC) [65]. EBAC's method does not provide a bare nucleon mass. *Row-3*: Bare masses inferred from the coupled-channels analysis described in Ref. [67], which describes the Roper resonance as dynamically-generated. In both these rows, "..." indicates states not found in the analysis. A visual comparison of these results is presented in Fig. 7.

	m_N	m_{N^*}	$m_{N\frac{1}{2}^-}$	$m_{N^*\frac{1}{2}^-}$	m_Δ	m_{Δ^*}	$m_{\Delta\frac{3}{2}^-}$	$m_{\Delta^*\frac{3}{2}^-}$
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EBAC		1.76	1.80	1.88	1.39	...	1.98	...
Jülich	1.24	none	2.05	1.92	1.46	...	2.25	...

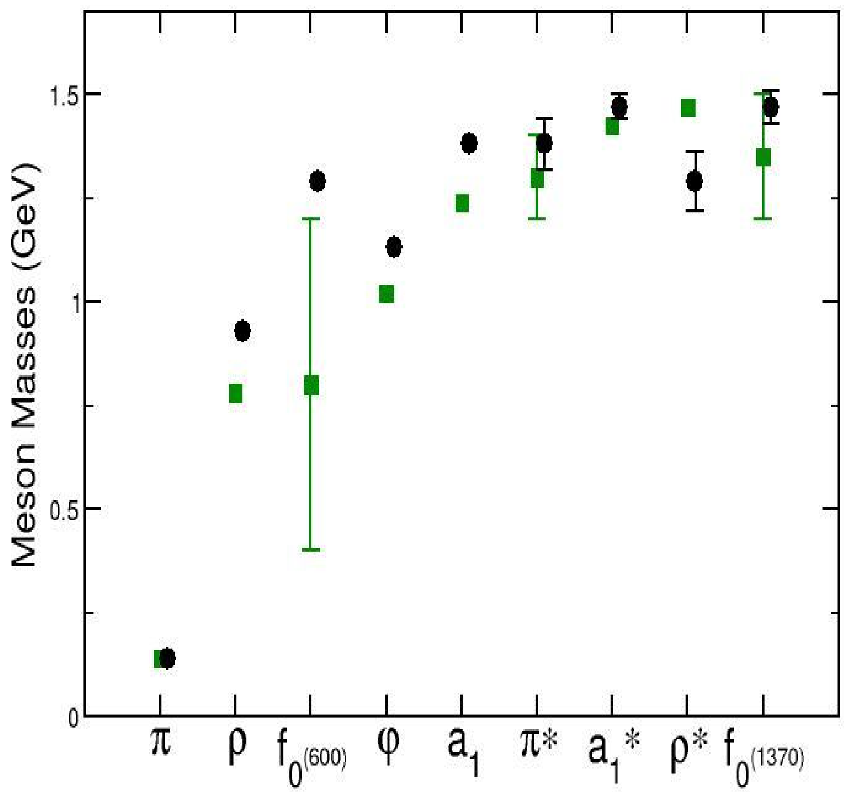
- In connection with EBAC's analysis, our predictions for the bare-masses agree within a rms-relative-error of 14%.
  - *Notably, EBAC does find a dressed-quark-core for the Roper resonance, at a mass which agrees with our prediction.*

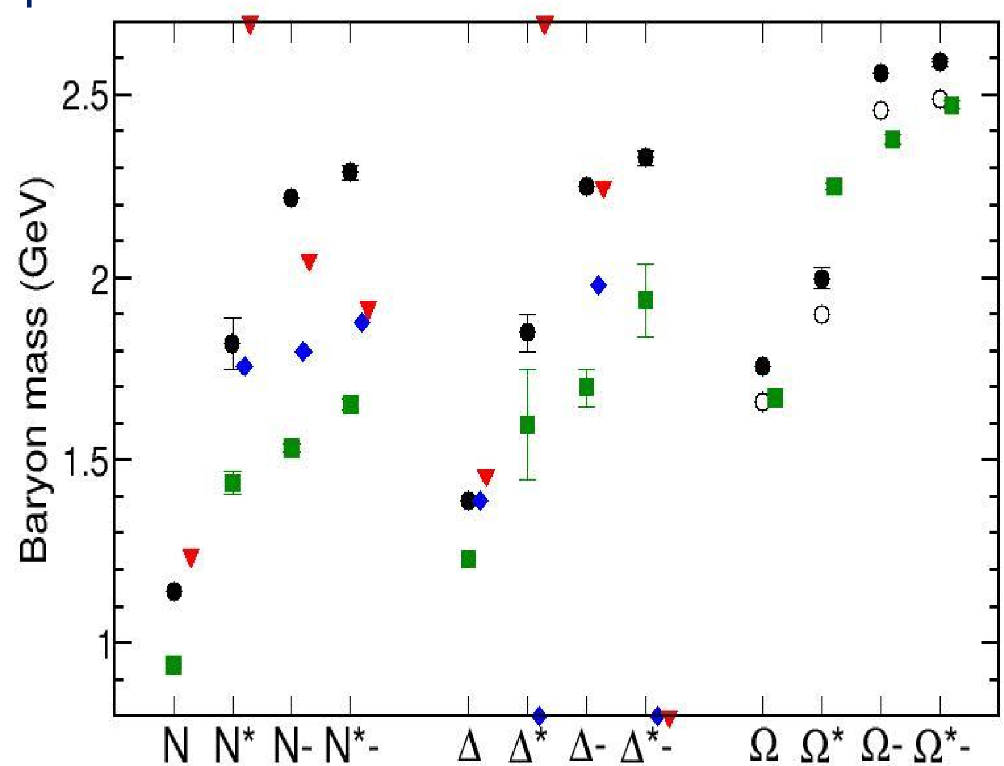
Legend:

- Particle Data Group
- H.L.L. Roberts *et al.*
- EBAC
- Jülich

Hadron Spectrum

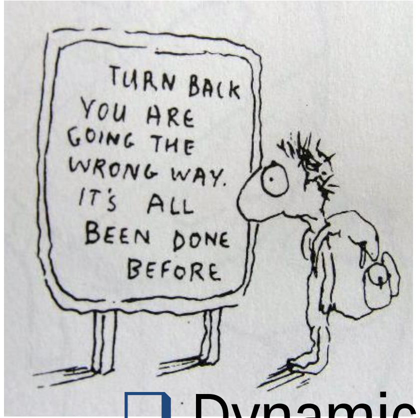
- o Symmetry-preserving unification of the computation of meson & baryon masses
- o rms-rel.err./deg-of-freedom = 13%
- o PDG values (almost) uniformly overestimated in both cases
  - room for the pseudoscalar meson cloud?!





Next steps…

- DSE treatment of static and electromagnetic properties of pseudoscalar and vector mesons, and scalar and axial-vector diquark correlations based upon a vector-vector contact-interaction.
- Basic motivation: need to document a comparison between the electromagnetic form factors of mesons and those diquarks which play a material role in nucleon structure. Important step toward a unified description of meson and baryon form factors based on a single interaction.
- Notable results:
  - Large degree of similarity between related meson and diquark form factors.
  - Zero in the ρ-meson electric form factor at $z_Q^{\rho} \approx \surd 6\, m_\rho$.
    Notably, $r_\rho\, z_Q^{\rho} \approx r_D\, z_Q^{D}$, where r_ρ, r_D are, respectively, the electric radii of the ρ-meson and deuteron.
- ***Ready now for nucleon elastic & nucleon→Roper transition form factors***



Epilogue

- Dynamical chiral symmetry breaking (DCSB) – *mass from nothing for 98% of visible matter* – is a reality
  - Expressed in $M(p^2)$, with observable signals in experiment

Confinement is almost Certainly the origin of DCSB

- Poincaré covariance

  Crucial in description of contemporary data

- Fully-self-consistent treatment of an interaction

  Essential if experimental data is *truly* to be *understood*.

- Dyson-Schwinger equations:
  - single framework, with IR model-input turned to advantage, *"almost unique in providing unambiguous path from a defined interaction → Confinement & DCSB → Masses → radii → form factors → distribution functions → etc."*

McLerran & Pisarski
arXiv:0706.2191 [hep-ph]